Understanding the Butterfly Strategy

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UNDERSTANDING THE BUTTERFLY STRATEGY

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Abstract

A butterfly, which is a combination of a barbell and a bullet, is one of the most common active fixed-income strategies used by practitioners. While being neutral to small parallel shifts of the yield curve, a butterfly is purposely exposed to specific bets on particular changes of the yield curve. There exist four different types of butterflies, the cash-and $duration neutral weighting butterfly, the fifty-fifty weighting regression, the regression weighting butterfly and the maturity weighting butterfly. In this paper, we show that they generate a positive pay-off when the particular flattening or steepening move of the yield curve they were structured for capturing occurs. We also argue that one suitable way to detect the opportunity to enter a specific butterfly is to use spread indicators. Finally, we show that the curvature $duration obtained from the Nelson and Siegel (1987) model can be used to measure the curvature risk of this strategy.

Keywords : fixed-income portfolio management, active strategy, butterfly, slope movement, spread indicators, curvature risk.

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UNDERSTANDING THE BUTTERFLY STRATEGY

INTRODUCTION

A *butterfly* is one of the most common fixed-income active strategies used by practitioners.[4] It is the combination of a barbell[5] (called *the wings of the butterfly*) and a bullet[6] (called *the body of the butterfly*). The purpose of the trade is to adjust the weights of these components so that the transaction is cash-neutral and has a $duration equal to zero. The latter property guarantees a quasi-perfect interest-rate neutrality when only small parallel shifts affect the yield curve. Besides, the butterfly, which is usually structured so as to display a positive convexity, generates a positive gain if large parallel shifts occur. On the other hand, as we let the yield curve be affected by more complex movements than parallel shifts, including slope and curvature movements, the performance of the strategy can be drastically impacted. It is in general fairly complex to know under which exact market conditions a given butterfly generates positive or negative pay-offs when all these possible movements are accounted for. There actually exist many different kinds of butterflies (some of which are not cash-neutral), which are structured so as to generate a positive pay-off in case a particular move of the yield curve occurs. Finally, we address the question of measuring the performance and the risk of a butterfly.

[4] See Choudry (2001) for a description of this strategy including in particular an example of a butterfly analysis on Bloomberg.

[5] A barbell portfolio is constructed by concentrating investments on the short-term and the long-term ends of the yield curve.

[6] A bullet portfolio is constructed by concentrating investments on a particular maturity of the yield curve.

1. A CONVEX TRADE

When only parallel shifts affect the yield curve, the strategy is structured so as to have a positive convexity. The investor is then certain to enjoy a positive pay-off if the yield curve is affected by a positive or a negative parallel shift. This point is illustrated in the following example.

Example 1 *We consider three bonds with short, medium and long maturities whose features are summarized in the following table.*

Maturity	*Coupon Rate*	*YTM*	*Bond Price*	*$Duration*	*Quantity*
2 Years	*5%*	*5%*	*100*	*185.9*	q_s
5 Years	*5%*	*5%*	*100*	*432.9*	*-1,000*
10 Years	*5%*	*5%*	*100*	*772.2*	q_l

These bonds are hypothetical bonds assumed to be default-risk free.

The face value of bonds is normalized to be $100, YTM stands for yield-to-maturity, bond prices are dirty prices and we assume a flat yield-to-maturity curve in this example. We structure a butterfly in the following way:

- *we sell 1,000 5-year maturity bonds*
- *we buy q_s 2-year maturity bonds and q_l 10-year maturity bonds*

The quantities q_s and q_l are determined so that the butterfly is cash and $duration neutral, i.e., we impose that they satisfy the following system:

$$\begin{cases} (q_s \times 185.9) + (q_l \times 772.2) = 1{,}000 \times 432.9 \\ (q_s \times 100) + (q_l \times 100) = 1{,}000 \times 100 \end{cases}$$

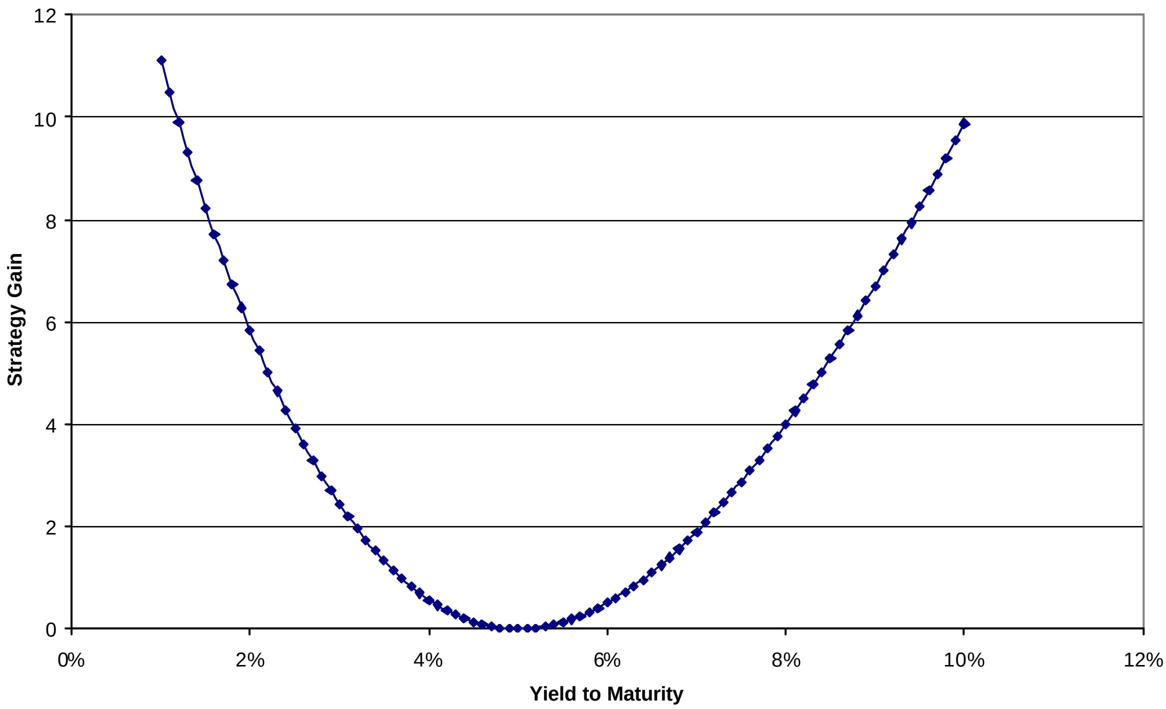
whose solution is

$$\begin{pmatrix} q_s \\ q_l \end{pmatrix} = \begin{pmatrix} 185.9 & 772.2 \\ 100 & 100 \end{pmatrix}^{-1} \times \begin{pmatrix} 432{,}900 \\ 100{,}000 \end{pmatrix} = \begin{pmatrix} 578.65 \\ 421.35 \end{pmatrix}$$

Of course, in a real market situation, we would buy 579 2-year maturity bonds and 421 10-year maturity bonds.[7]*. We now draw the profile of the strategy gain depending on the value of the yield to maturity (see* **Figure 1***).*

[7] On the market you actually buy bonds in terms of amount and not in terms of number of securities. You will or not round the amount you want to buy, depending on the minimum amount of the security that can be purchased. Here, for simplicity, the quantity of bonds is measured in terms of number.

Figure 1. Profile of the P&L's butterfly strategy depending on the value of the yield to maturity



The butterfly has a positive convexity. Whatever the value of the yield to maturity, the strategy always generates a gain. This gain is all the more substantial as the yield to maturity reaches a level further away from 5%. The gain has a convex profile with a perfect symmetry around the 5% X-axis. For example, the total return reaches $57 when the yield to maturity is 4%.

We know, however, that the yield curve is potentially affected by many other movements than parallel shifts. These include in particular pure slope and curvature movements, as well as combinations of level, slope and curvature movements[8]. It is in general fairly complex to know under what exact market conditions a given butterfly might generate a positive or a negative pay-off when all these possible movements are accounted for[9]. Some butterflies are structured so as to pay off if a particular move of the yield curve occurs.

2. DIFFERENT KINDS OF BUTTERFLIES

While a feature common to all butterflies is that they always have a $duration equal to zero, they actually come in many very different shapes and forms that we now examine in details. In the case of a standard butterfly, the barbell is a combination of a short-term and a long-term bonds and the bullet is typically a medium-term bond. a , the quantity of the medium-term bond in the portfolio, is defined at date 0 by the investor.

Maturity	Bond Price	Quantity	$Duration
Short	P_c	q_s	D_s
Medium	P_m	$q_m = a$	D_m
Long	P_l	q_l	D_l

[8] See for example Litterman and Scheinkman (1991).
[9] See Mann and Ramanlal (1997).

2.1 Cash and $Duration Neutral Weighting

The idea is to adjust the weights so that the transaction has a zero $duration, and the initial net cost of the portfolio is also zero, which can be written as

$$\begin{cases} q_s D_s + q_l D_l + a D_m = 0 \\ q_s P_s + q_l P_l + a P_m = 0 \end{cases} \qquad (1)$$

Solving this linear system yields the quantities q_s and q_l to hold in the short-term and the long-term bonds, respectively.

Example 2 *We consider three bonds with the following features*

Maturity	YTM	Bond Price	Quantity	$Duration
2 Years	4.5%	100.936	q_s	188.6
5 Years	5.5%	97.865	-10,000	421.17
10 Years	6%	92.64	q_l	701.14

These bonds are again hypothetical bonds of the stated characteristics and are assumed to be default-risk free. We structure a butterfly in the following way:

- *we sell 10,000 5-year maturity bonds*
- *we buy q_s 2-year maturity bonds and q_l 10-year maturity bonds*

Quantities q_s and q_l are determined so that the butterfly satisfies the system (1):

$$\begin{cases} (q_s \times 188.6) + (q_l \times 701.14) = (10{,}000 \times 421.17) \\ (q_s \times 100.936) + (q_l \times 92.64) = (10{,}000 \times 97.865) \end{cases}$$

Solving the system we obtain q_s and q_l

$$\begin{pmatrix} q_s \\ q_l \end{pmatrix} = \begin{pmatrix} 188.6 & 701.14 \\ 100.936 & 92.64 \end{pmatrix}^{-1} \times \begin{pmatrix} 4{,}211{,}734 \\ 978{,}649 \end{pmatrix} = \begin{pmatrix} 5{,}553.5 \\ 4{,}513.1 \end{pmatrix}$$

Some strategies do not require a zero initial cash-flow. In this case, there is an initial cost of financing. Three classic strategies are the fifty-fifty weighting butterfly, the regression weighting butterfly as first described by Grieves (1999) and the maturity weighting butterfly.

2.2 Fifty-Fifty Weighting

The idea is to adjust the weights so that the transaction has a zero $duration and the same $duration on each wing so as to satisfy the two following equations

$$\begin{cases} q_s D_s + q_l D_l + a D_m = 0 \\ q_s D_s = q_l D_l = \dfrac{-a D_m}{2} \end{cases} \qquad (2)$$

The aim of this butterfly is to make the trade neutral to some small steepening and flattening movements. In terms of yield-to-maturity (YTM), if the spread change between the body and the short wing is equal to the spread change between the long wing and the body, a fifty-fifty weighting butterfly is neutral to such curve movements. Then, for a steepening scenario "-30/0/30", which means that the short wing YTM decreases by 30 bps and the long wing YTM increases by 30 bps while the body YTM does not move, the trade is quasi curve neutral. The same would apply for a flattening scenario "30/0/-30".

Example 3 *We consider the same components of the butterfly as in* **Example 2**.

Quantities q_s *and* q_l *are determined so that the butterfly satisfies the system (2):*

$$\begin{cases} (q_s \times 188.6) + (q_l \times 701.14) = (10{,}000 \times 421.17) \\ (q_s \times 188.6) - (q_l \times 701.14) = 0 \end{cases}$$

Solving the system we obtain q_s *and* q_l

$$\begin{pmatrix} q_s \\ q_l \end{pmatrix} = \begin{pmatrix} 188.6 & 701.14 \\ 188.6 & -701.14 \end{pmatrix}^{-1} \times \begin{pmatrix} 4{,}211{,}734 \\ 0 \end{pmatrix} = \begin{pmatrix} 11{,}165.7 \\ 3{,}003.5 \end{pmatrix}$$

The fifty-fifty weighting butterfly is not cash neutral. In the example above, the portfolio manager has to pay \$426,623 and if he carries the position during one day he will have to support a financing cost equal to \$46 assuming a one day short rate equal to 4%.

2.3 Regression Weighting

The idea is to adjust the weights so that the transaction has a zero \$duration, and so as to satisfy the two following equations

$$\begin{cases} q_s D_s + q_l D_l + a D_m = 0 \\ q_s D_s \times (1/b) = q_l D_l \end{cases} \qquad (3)$$

As short-term rates are much more volatile than long-term rates, we normally expect that the short wing moves more from the body than the long wing. This stylized fact motivates the introduction of a coefficient b obtained by regressing changes in the spread between the long wing and the body on changes in the spread between the body and the short wing. This coefficient is of course dependent on the data frequency used (daily, weekly or monthly changes). Assuming that we obtain a value of, say, 0.5 for the regression coefficient, it means that for a 20 bps spread change between the body and the short wing, we obtain on average a 10 bps spread change between the long wing and the body. Then for a steepening scenario "-30/0/15", which means that the short wing YTM decreases by 30 bps, the body YTM does not move and the long wing YTM increases by 15 bps, or a flattening scenario "30/0/-15", the trade is quasi curve neutral. Note finally that the fifty-fifty weighting butterfly is equivalent to a regression weighting butterfly with a regression coefficient equal to 1.

Example 4 *We consider the same components of the butterfly as in example 2.*

Quantities q_s and q_l are determined so that the butterfly satisfies the system (3):

$$\begin{cases} (q_s \times 188.6) + (q_l \times 701.14) = (10{,}000 \times 421.17) \\ (q_s \times 188.6) - (0.5 \times q_l \times 701.14) = 0 \end{cases}$$

Solving the system we obtain q_s and q_l

$$\begin{pmatrix} q_s \\ q_l \end{pmatrix} = \begin{pmatrix} 188.6 & 701.14 \\ 188.6 & -350.57 \end{pmatrix}^{-1} \times \begin{pmatrix} 4{,}211{,}734 \\ 0 \end{pmatrix} = \begin{pmatrix} 7{,}443.8 \\ 4{,}004.7 \end{pmatrix}$$

In the example above, the portfolio manager has to pay \$143,695 and if he carries the position during one day he will have to incur a financing cost equal to \$15 assuming a one day short rate equal to 4%.

2.4 Maturity-Weighting

The idea is to adjust the weights so that the transaction has a zero \$duration and so as to satisfy the three following equations

$$\begin{cases} q_s D_s + q_l D_l + a D_m = 0 \\ q_s D_s = -a\left(\dfrac{M_m - M_s}{M_l - M_s}\right) D_m \\ q_l D_l = -a\left(\dfrac{M_l - M_m}{M_l - M_s}\right) D_m \end{cases} \qquad (4)$$

where M_s, M_m and M_l are the maturities of the short-term, the medium-term and the long-term bonds, respectively.

Maturity-weighting butterflies are structured similarly to the regression butterflies, but instead of searching for a regression coefficient b that is dependent on historical data, the idea is to weight each wing of the butterfly with a coefficient depending on the maturities of the three bonds. In fact using equation (4) we show that

$$q_s D_s = \left(\frac{M_m - M_s}{M_l - M_m}\right) q_l D_l$$

Finally a maturity weighting butterfly is equivalent to a regression weighting butterfly with a regression coefficient equal to $b = \left(\dfrac{M_m - M_s}{M_l - M_m}\right)$

Example 5 *We consider the same components of the butterfly as in* **Example 2**.

Quantities q_s *and* q_l *are determined so that the butterfly satisfies the system (4):*

$$\begin{cases} (q_s \times 188.6) + (q_l \times 701.14) = (10{,}000 \times 421.17) \\ (q_s \times 188.6) = (10{,}000 \times (3/8) \times 421.17) \end{cases}$$

Solving the system we obtain q_s *and* q_l

$$\begin{pmatrix} q_s \\ q_l \end{pmatrix} = \begin{pmatrix} 188.6 & 701.14 \\ 188.6 & 0 \end{pmatrix}^{-1} \times \begin{pmatrix} 4{,}211{,}734 \\ 1{,}579{,}400 \end{pmatrix} = \begin{pmatrix} 8{,}374.3 \\ 3{,}754.4 \end{pmatrix}$$

In the example above, the portfolio manager has to pay $214,427 and if he carries the position during one day he will have to support a financing cost equal to $23 assuming a one day short rate equal to 4%.

3. HOW TO MEASURE THE PERFORMANCE AND THE RISK OF A BUTTERFLY ?

There are two possible ways of detecting interesting opportunities for a butterfly strategy. The first indicator, the total return indicator, may also be applied to other types of strategies. The second indicator is based upon an analysis of historical spreads.

3.1 Total Return Measure

In an attempt to measure the abnormal performance of a given strategy with respect to another given strategy for a specific scenario of yield curve evolution, one needs to perform a total return analysis. This implies taking into account the profit in terms of price changes, interest paid, and reinvestment on interest and principal paid. The total return in $ from date t to t+dt is given by:

Total Return in $ = (sell price at date t+dt - buy price at date t + received coupons from date t to t+dt + interest gain from reinvested payments from date t and t+dt)

When the butterfly generates a non-zero initial cash-flow, we calculate the net total return in $ by substracting the financing cost from the total return in $:

Net Total Return in $ = Total Return in $ - Financing Cost in $

Example 6 *Taking into account the four kinds of butterfly strategies, we consider again the same components of the butterfly as in* **Example 2**.

We always structure the butterfly so as to sell the body and buy the wings, and assume seven different movements of the term structure:

- *No movement ("Unch" for unchanged)*
- *Parallel movements with a uniform change of +20 bps or -20 bps for the three YTMs*

- *Steepening and flattening movements in which the curve rotates around the body; for example "-30/0/30" meaning that the short wing YTM decreases by 30 bps, the body YTM does not move and the long wing YTM increases by 30 bps.*

Table 1 *displays net total returns in $ for the four different butterflies carried for only one day. Besides we assume that the cost of carry is 4%.*

Table 1. Overnight Butterfly Trades (sell the body, buy the wings) - Daily Net Total Return in $

Kinds of Butterfly	Unch	+20	-20	-30/0/30	30/0/-30	-30/0/15	30/0/-15
Cash- and $Duration-Neutral	-9	11	11	-6,214	6,495	-1,569	1,646
Fifty-Fifty Weighting	-9	-1	-5	140	116	3,192	-3,110
Regression Weighting	-9	7	6	-4,087	4,347	35	44
Maturity Weighting	-9	5	3	-3,040	3,289	824	-744

For an unchanged curve, the net total return in $ for each of the butterflies is very low as we may expect since the position is carried out during just one day.

In the second and third columns of the table, we increase and decrease, respectively, all three YTMs by 20 bps. Because the four strategies are $duration neutral, the net returns are very near from zero. The wings, which exhibit larger convexity, outperform the body for parallel shifts, except for the fifty-fifty weighting because of the cost of carry.

The fourth to seventh columns of the table show the results for different steepening and flattening scenarios. The net total return is very different from one butterfly to another. We note that:

- *The cash and $duration neutral weighting butterfly has a negative return for a steepening and positive for a flattening. This is because the major part of the $duration of the trade is in the long wing. When the move of the long wing YTM goes from 30 bps to 15 bps, the net return increases from -$6,214 to $1,569 and inversely decreases from $6,495 to $1,646 when the long wing YTM goes from -30 bps to -15 bps.*

- *For the fifty-fifty weighting butterfly, when the changes between the body and the short wing on the one hand, and between the long wing and the body on the other hand, are equal, the net total return is very close to zero. This is because the butterfly is structured so as to have the same $duration in each wing. Besides we note that returns are positive because of the difference of convexity between the body and the wings. The fifty-fifty weighting butterfly has a positive return for a steepening and a negative return for a flattening.*

- *As expected, the regression weighting butterfly with a regression coefficient equal to 0.5 is quasi curve neutral to the two scenarios for which it was structured ("-30/0/15" and "30/0/-15"). Returns are positive because of a difference of convexity between the body and the wings. It has a negative return for the steepening scenario "-30/0/30" as it has a positive return for the flattening scenario "30/0/-30" because most of the $duration is in the long wing.*

- *The maturity weighting scenario has about the same profile as the regression weighting butterfly. In fact it corresponds to a regression weighting butterfly with a regression coefficient equal to 0.6 ((5-2)/(10-5)). For a flattening scenario ("30/0/-18") and a steepening scenario ("-30/0/18"), it would be curve neutral.*

3.2 Spread Measures

Spread measures provide very good estimates of total returns in dollars. This indicator applies to all kinds of butterfly except for the cash- and $duration-neutral combination.

- For a fifty-fifty butterfly, the approximate total return in $ is given by

$$\text{Total Return in \$} \approx a D_m \Delta R_m + q_s D_s \Delta R_s + q_l D_l \Delta R_l$$

Which translates into, using equation (2)

$$\text{Total Return in \$} \approx a D_m \left[\Delta R_m - \left(\frac{\Delta R_s + \Delta R_l}{2} \right) \right]$$

This is used to determine the following spread

$$R_m - \left(\frac{R_s + R_l}{2} \right)$$

- For a regression butterfly, the approximative total return in $ is given by

$$\text{Total Return in \$} \approx a D_m \Delta R_m + q_s D_s \Delta R_s + q_l D_l \Delta R_l$$

From equation (3), we obtain

$$\text{Total Return in \$} \approx a D_m \left[\Delta R_m - \left(\frac{b}{b+1} \right) \Delta R_s - \left(\frac{1}{b+1} \right) \Delta R_l \right]$$

which is used to determine the following spread

$$R_m - \left(\frac{b}{b+1} \right) R_s - \left(\frac{1}{b+1} \right) R_l$$

- For a strategy with a maturity weighted butterfly, the approximative total return in $ is also given by

$$\text{Total Return in \$} \approx a D_m \Delta R_m + q_s D_s \Delta R_s + q_l D_l \Delta R_l$$

From equation (4) we obtain

$$\text{Total Return in \$} \approx a D_m \left[\Delta R_m - \left(\frac{M_m - M_s}{M_l - M_s} \right) \Delta R_s - \left(\frac{M_l - M_m}{M_l - M_s} \right) \Delta R_l \right]$$

which yields the following spread

$$R_m - \left(\frac{M_m - M_s}{M_l - M_s} \right) R_s - \left(\frac{M_l - M_m}{M_l - M_s} \right) R_l$$

Using these three spread indicators, we calculate the approximative total returns in $ for the butterflies examined in the previous example. Results are summarized in **Table 2**. From a comparison to the total returns in $ obtained in the previous table, we can see that spread indicators provide a very accurate estimate of the total returns in $.

Table 2. Overnight Butterfly Trades (sell the body, buy the wings)
Approximative Total Return in $ Using Spread Indicators

Kinds of Butterfly	Unch	+20	-20	-30/0/30	30/0/-30	-30/0/15	30/0/-15
Fifty-Fifty Weighting	0	0	0	0	0	3,159	-3,159
Regression Weighting	0	0	0	-4,212	4,212	0	0
Maturity Weighting	0	0	0	-3,159	3,159	790	-790

A historical analysis of these spreads gives an indication of the highest or lowest values, which may be used as indicators of opportunities to enter a butterfly strategy. Note that this spread analysis does not take into account the effect of received coupons and financing costs of the trades.

4. LEVEL, SLOPE AND CURVATURE $DURATION RISK MEASURES

One way to measure the sensitivity of a butterfly to interest rate risk is to compute the level, slope and curvature $durations in the Nelson and Siegel (1987) model, which provides a popular parametrization of the zero-coupon yield curve. The model for the yield curve contains four parameters and allows one to obtain the standard increasing, decreasing, flat and inverted shapes. The model is specified as follows

$$R^C(0,q) = b_0 + b_1\left[\frac{1-\exp(-q/t)}{q/t}\right] + b_2\left[\frac{1-\exp(-q/t)}{q/t} - \exp(-q/t)\right] \qquad (5)$$

where:

$R^C(0,q)$ is the continuously compounded zero-coupon rate at time zero with maturity q .

b_0 is the limit of $R^C(0,q)$ as q goes to infinity. In practice, b_0 should be regarded as a long-term interest rate.

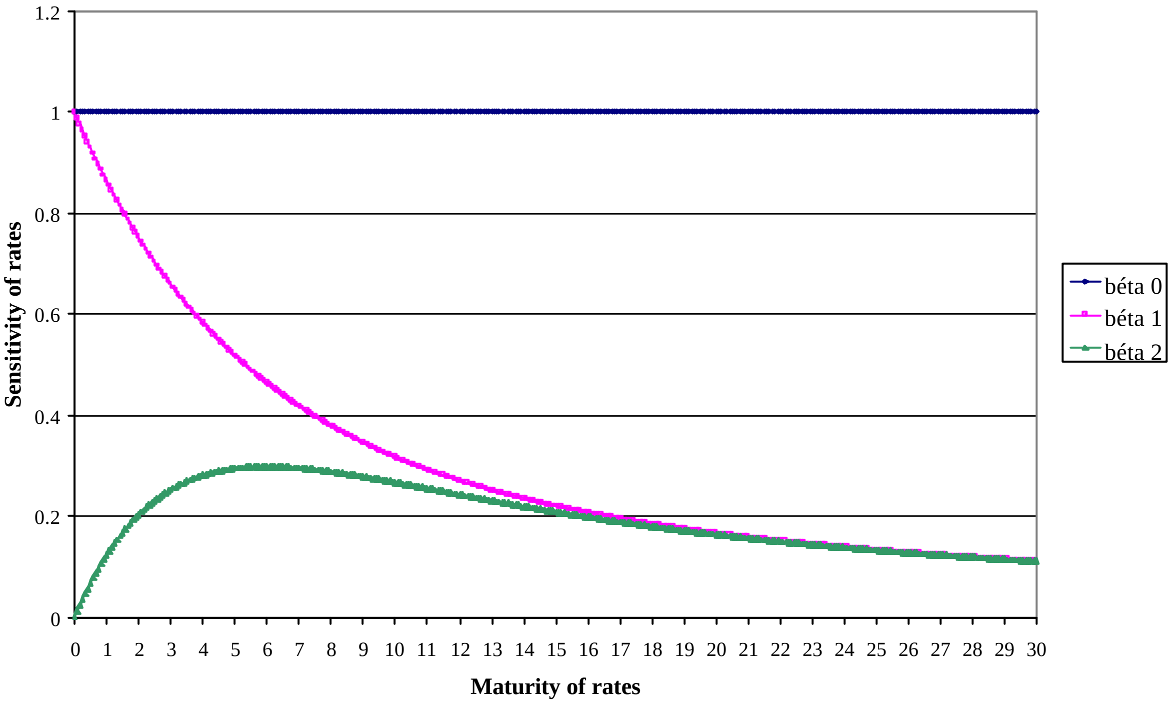
b_1 is the limit of $b_0 - R^C(0,q)$ as q goes to 0. In practice, b_1 should be regarded as the short to long term spread.

b_2 is a curvature parameter.

t is a scale parameter that measures the rate at which the short-term and medium-term components decay to zero.

The advantage of this model is that the three parameters b_0, b_1 and b_2 can directly be related to parallel shifts, slope shifts and curvature changes in the yield curve, as illustrated by **Figure 2** which shows the sensitivity $S_i = \frac{\partial R^C(0,q)}{\partial b_i}$ of zero-coupon rates to each parameter b_i for i=0,1,2. The level factor S_0 is constant across maturities. The slope factor S_1 is largest for short maturities and decreases exponentially toward zero as maturity increases. Starting from zero for short maturities, the curvature factor S_2 reaches a maximum at the middle of maturity spectrum and then decreases to zero for longer maturities.

Figure 2. Sensitivity of the zero-coupon rates to the parameters of the Nelson-Siegel functional form



These sensitivities are obtained by fixing parameter values equal to b_0 =7%, b_1 = -2%, b_2 =1% and $1/t$ =0.3.

The price P_t at date t of a butterfly (sell the body and buy the wings) is the sum of its n future discounted cash-flows C_i multiplied by the amount invested q_i (for example, q_i is –1,000 for cash-flows of the body if we sell 1,000 of the medium-term bond). Some of these cash-flows are of course negative because we sell the body. The price is expressed as follows

$$P_t = \sum_{i=1}^{n} q_i . C_i . e^{-q_i R^C(t,q_i)}$$

Using equation (5), the butterfly level, slope and curvature $durations, denoted respectively by D_0, D_1 and D_2, are given by

$$\begin{cases} D_{0,t} = \dfrac{\partial P_t}{\partial b_0} = -\sum q_i . q_i C_i e^{-q_i R^C(t,q_i)} \\ D_{1,t} = \dfrac{\partial P_t}{\partial b_1} = -\sum q_i . q_i \left[\dfrac{1-\exp(-q_i/t)}{q_i/t} \right] C_i e^{-q_i R^C(t,q_i)} \\ D_{2,t} = \dfrac{\partial P_t}{\partial b_2} = -\sum q_i . q_i \left[\dfrac{1-\exp(-q_i/t)}{q_i/t} - \exp(-q_i/t) \right] C_i e^{-q_i R^C(t,q_i)} \end{cases} \quad (6)$$

At any date t, from a derivation of the zero-coupon yield curve based on this model[10], one can compute the level, slope and curvature $durations of the butterfly. Of course, $duration D_0 is expected to be very small because the butterfly is structured so as to be neutral to small parallel shifts. $duration D_1 provides the exposure of the trade to the slope factor. Finally, $duration D_2 quantifies the curvature risk of the butterfly that can be neutralised by proper hedging.

Example 7 *We compute below the level, slope and curvature $durations of a butterfly in the Nelson and Siegel (1987) model.*

At date t=0, the values of the parameters are the following

b_0	b_1	b_2	t
8%	-3%	-1%	3

We consider three hypothetical default risk-free bonds with the following features. The face value of bonds is $100. We structure a cash-and neutral butterfly by selling the body and buying the wings. We compute the level, slope and curvature $durations of the butterfly from equations (6).

	Maturity	Coupon	Quantity	Price	Level D_0	Slope D_1	Curvature D_2
Bond 1	2 years	5%	472	98.627	-192.51	-141.08	-41.28
Bond 2	7 years	5%	-1,000	90.786	-545.42	-224.78	-156.73
Bond 3	15 years	5%	556	79.606	-812.61	-207.2	-172.03
Butterfly				0$	2,744	42,987	41,041

The interpretation of these results is straightforward. For example, based on the butterfly slope $duration D_1, one expects a 0.1% increase of the b_1 parameter to increase the value of the butterfly by 42.987$ (42,987$\times$0.1%). As expected, we see in particular that the level $duration is small compared to the slope and curvature $durations.

[10] See for example Martellini and Priaulet (2000).

CONCLUSION

While being neutral to small parallel shifts of the yield curve, the purpose of a butterfly strategy is to take specific bets on particular changes of the yield curve. There exist four different types of butterflies, the cash-and $duration neutral weighting butterfly, the fifty-fifty weighting regression, the regression weighting butterfly and the maturity weighting butterfly. In this paper, we show that they have a positive pay-off in case the particular flattening or steepening move of the yield curve they were structured for occurs. We also argue that spread indicators offer a convenient way of detecting the opportunity to enter a specific butterfly.

Besides, one suitable method to hedge the risk of a butterfly is to use the Nelson and Siegel (1987) model. The idea is to compute the level, slope and curvature durations of the butterfly in this model, and then to construct semi-hedged strategies[11]. A portfolio manager structuring for example a fifty-fifty weighting butterfly (by selling the body and buying the wings) is then able to take a particular bet on a steepening move of the yield curve while being hedged against the curvature risk.

[11] See Martellini, Priaulet and Priaulet (2002).

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