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Strips: Arbitraging the Eurodollar Cash and Futures Markets
by Larry Grannan

In December 1981, Chicago Mercantile Exchange (CME) introduced a futures contract based on 3-month Eurodollar (ED) interest rates. In the twenty-some years since its inception, this contract has become one of the most versatile trading and hedging vehicles offered on the listed markets. The contract represents a $1,000,000, 3-month London Interbank Offered Rate (LIBOR) deposit. CME Eurodollar futures are cash-settled; therefore, there is no delivery of a cash instrument upon expiration because cash Eurodollar time deposits are not transferable.

Of the contract's many uses, one of the most significant has been by arbitrageurs in the money markets who often combine cash and futures positions to create a synthetic instrument called a "strip." Many of the trades in the quarterly Eurodollar futures contracts are devoted to these strategies. A strip can mean combining cash deposits (borrowings) with a long (short) position in the futures contracts. Such a trade is initiated when it is determined that the trader can lock in a higher return or a lower borrowing cost than is otherwise available in a cash-only money market transaction. The term "strip" is derived from the practice of using two or more consecutive quarterly futures expirations in combination with a Eurodollar cash position.

The traders must determine for themselves whether the spread between the strip interest rate and the cash-only transaction is wide enough to make such a trade worthwhile. Below, we will calculate a strip rate using actual market rates, but first it might be beneficial to review how Eurodollar futures are priced.

For example, on April 19, 2000, the September 2000 Eurodollar futures traded at an index price of 93.21. CME calculates this price by subtracting the ED interest rate (in this case, 6.79%) from 100. Although the contract is referred to as 3-month Eurodollar futures, it did not match that day's prevailing cash 3-month LIBOR rate of 6.31%. The futures contract is meant to represent a 3-month implied forward rate on the date of the contract's expiration on September 18, 2000, or 152 days hence.

To illustrate how closely the Eurodollar futures will track the implied forward rates, it might be useful to derive a 3-month forward Eurodollar rate. We can then compare this rate to that of the September ED futures contract, whose rate is meant to match that of a 3-month cash deposit which settles on September 20. The futures contract is based on a 90-day rate (and therefore a constant $25 per $.01 change in price), even though the period actually covered by the futures contract may very well be different. Recognition of this potential mismatch in basis point values between the futures contract and the deposit to be hedged will influence the number of contracts used to hedge a given exposure.

We also assume in this example that deposit transactions settle two business days after trades are agreed, and that deposits mature on the same calendar date as the settlement, "n" months in the future. Counting days in this manner from April 19, 2000, the September 2000 ED futures expire (or "fix") in 152 days (on September 18), and settle in 154 days (on September 20). The interest rate on the 3-month deposit that the futures contract represents is fixed on September 18, actual funds are received on September 20, and the deposit matures 91 days later on December 20 — 245 days from April 19.

We assume that an investor should be indifferent between investing for 245 days, and investing for 154 days and rolling the proceeds for 91 days. If this is the case, "fair value" for the September futures contract is the 91-day rate at which a 154-day investment could be rolled at maturity so that its return equals that of a 245-day investment made on April 19. LIBOR cash rates are usually quoted for overnight, one week, and then in maturities month-to-month up to one year. The 154- and 245-day periods may seem like unusual durations for deposits, but these would correspond to 5-month and 8-month LIBOR rates. However, the most readily available rates from banks are usually quarterly rates such as 3-, 6-, 9- and 12-month rates. Therefore, if there were no readily available markets for these 154- and 245-day maturities, a linear interpolation could be performed from the most readily available Eurodollar rates. Although they might not represent the exact rates that a bank might offer, they do facilitate the derivation of forward rates. A 154-day rate could be derived from a 6-month rate of 6.52% and a 3-month rate of 6.31% as follows:

6 month = 180 days = .0652
3 month = 90 days = .0631
Total Difference = .002100

Difference/day = .0021/90 days = .000023
154-day rate = 90 day rate + (.000023 x 64 days)
= .063100 + .001472
= .064572 ≈ 6.46%

Similarly, a 245-day rate can be interpolated from the 6-month rate of 6.52% and a 9-month rate of 6.72875% to arrive at 6.67%. The forward rate can be derived using the following break-even formula where r* is the implied forward rate.

$$[1+.0646\ (154/360)][1+r^*(91/360)]=[1+.0667\ (245/360)]$$

The forward rate of r* is 6.84%, which is extremely close to the rate on April 19 for the September Eurodollar futures contract of 6.79% (93.21). At times these forward rates are quite close to the futures prices while, at other times, market dynamics may cause the futures prices to diverge dramatically from cash prices. When the futures contract expires, the settlement price is a function of cash LIBOR rates on the last day of trading, so cash and futures rates converge as expiration approaches. Futures may look "cheap" or "expensive" to cash, and this can assist hedgers in determining which futures contract should be bought or sold. Arbitrageurs, usually large bank dealers, will enter the market to take advantage of these discrepancies by doing a strip transaction.

While most traders of strips will utilize computer software to identify profitable cash/futures spreads, the process of determining a strip rate is relatively simple, as the equation below shows.

$$\text{STRIP \%} = \{ [1+R_{cash} (n/360)]\ [1+R_{futures} (t/360)] - 1\}\ 360/n+t$$

This equation is the basic method for deriving a strip rate within one year. The maturity of the first cash deposit (or borrowing) is represented by "n." This first cash transaction is also known as the "front tail" or "stub," which is multiplied by a cash interest rate (Rcash) for that period.

The second part of the equation is the rate and tenure that the first futures contract represents. The figure "t" represents the time period, in days, between the settlement of the nearby and the next maturing quarterly futures contract. Following this would be further components representing other futures contracts in the strip. The trader can compare the approximation that the strip yield represents to an actual cash market rate to see if a combination of cash and futures will outperform a cash-only trade.

For example, we can derive a strip rate for the morning of April 19, 2000. We assume that a trader has acquired funds and can place them at LIBOR for a period of 11 months ending on March 20, 2001. The strip will involve placing funds in a 2-month cash deposit at LIBOR of 6.21% and purchasing June 2000 Eurodollar futures contracts at 93.395 (6.605%), September 2000 at 93.21 (6.79%), and December 2000 at 93.05 (6.95%). The mechanics of the long strip involve placing the cash deposit for a period that runs up to two days following the expiration of the first long futures contract — in this case, June 21, 2000, 63 days hence. In theory, after the first futures contracts are sold off, the cash deposit will be rolled over for a period corresponding to the length of time between the June settlement on June 21, and the September settlement on September 20. It is assumed that this process continues throughout the expiration of the December contract when funds will be placed in the cash market until March 21, 2001. In the long strip, the whole idea is that you create a synthetic asset by using futures. The funds are initially placed in the cash markets but are rolled forward into new deposits as the futures mature. If the reinvestment rate declines, i.e., futures prices rise, this is offset by the futures gains. If interest rates rise, i.e., interest rate futures prices fall, and the futures losses are offset by the higher reinvestment rate cash deposits.

Let us consider our proposed transaction.
In the cash market, 11-month LIBOR rates are 6.80375% on April 19, 2000. Below is the derivation of the strip rate of 6.84%.

6.84% =
{ [1+(.0621) 63/360][1+(.06605)91/360][1+(.0679)91/360]
[1+ (.0695)91/360] – 1} 360/63+91+91+91

The decision to employ a strip depends upon several factors. In the case above, the higher strip yield might justify the transaction. It would, however, also depend on how much basis risk the investor would accept and whether the premium received by the strip would compensate for the transaction cost of futures commissions. The above equation with its futures maturities of 91, 91, and 91 days suggests a uniform number of days between each settlement. However, this is not always the case. Depending on the year, these periods could run from 90 to 98 days. This illustrates the risk that the investor may not be able to find deposits whose maturities exactly match futures expirations. It is because of risks such as this that many in the markets may not put on such a trade unless a premium of 25 to 30 basis points could be guaranteed. The markets have been heavily arbitraged in recent years and trades such as this are no longer "easy pickings."

There are other concerns for investors who might consider such a trade. In our example, we restricted ourselves to LIBOR for our deposit rates. But money can be borrowed and deposits made at rates such as LIBID and LIMEAN, with spreads above or below these levels depending upon the customer's creditworthiness. While the example of an 11-month deposit versus a strip was good for illustrative purposes, the end date for such a transaction frequently will not match the implied settlement date of a Eurodollar futures contract such as March 21, 2001. However, we must be able to do the strip transaction starting any day and for any period of time to match a cash rate, even though there may not be a convenient end date matching a CME expiration.

We can address this problem by employing the same strategy for 12 months instead of only 11. Assume the investor received funds for one year until April 19, 2001. The investor could end up buying additional March 2001 Eurodollar futures contracts on April 19, 2000 at 93.01 (6.99%).The previous strip equation would be amended to include the component for March 2001 futures of [1+(.0699)(29/360)]360/336+29. This would give us a strip rate of 6.88% versus a cash one-year LIBOR rate of 6.84%.

The slightly higher strip yield means that this might be preferable to the straight cash deposit as a more attractive investment. The strip also presents us once again with the predicament of mismatching dates. The contract covers the period of March 21, 2001 to June 20, 2001. The period we want to cover is March 21 to April 19, a time span of 29 days, not the 91 covered by the futures contract. In this case, we are hedging a 29-day period with a derivative product that represents a 3-month duration. If a $100 million position were to be covered, we could try to achieve dollar equivalency with the futures contract by purchasing 32 of the March 2001 Eurodollar futures (100MM x 29/91) instead of approximately 100 that might be employed with the June, September and December expirations.

The number of contracts might well cover the face value and duration of the cash position, but it is an approximation that could be further fine-tuned. For example, the above case called for constructing a "tail." In addition, many hedgers will underweight the size of their hedge slightly as a function of the tenures of the hedge and underlying instrument as well as the cost of financing their margin flows. This adjustment in their hedge ratio accounts for interest earned on profits from positive margin flows and interest paid on outflows due to margin calls.

Cash U.S. Treasury Market Fundamentals and Basics of Eurodollar/Treasury Spreading

by John W. Labuszewski

This document is intended to provide an overview of the fundamentals of trading cash Treasury securities and how they can be spread against CME's Eurodollar futures. We assume only a cursory knowledge of coupon-bearing Treasury securities.

1. Coupon-Bearing Treasury Securities

U.S. Treasury bonds and notes represent a loan to the U.S. government. Bondholders are creditors rather than equity- or share-holders. The U.S. government agrees to repay the face or principal or par amount of the security at maturity, plus coupon interest at semi-annual intervals. Treasury securities are often considered "riskless" investments given that the "full faith and credit" of the U.S. government backs these securities.

The security buyer can either hold the bond until maturity, at which time the face value becomes due; or, the bond may be sold in the secondary markets prior to maturity. In the latter case, the investor recovers the market value of the bond, which may be more or less than its face value, depending upon prevailing yields. In the meantime, the investor receives semi-annual coupon payments every six months.

Suppose, for example, you purchase $5 million face value of the 5-3/8% note maturing in June 2000. This security pays half its stated coupon or 2.6875% of par on each six-month anniversary of its issue. Thus, you receive $134,375 semi-annually. Upon maturity in June 2000, the $5 million face value is re-paid and the note expires.

Price/Yield Relationship

A key factor governing the performance of bonds in the market is the relationship of yield and price movement. In general, as yields increase, bond prices will decline; as yields decline, prices rise. In a rising rate environment, bondholders will witness their principal value erode; in a declining rate environment, the market value of their bonds will increase.

IF Yields Rise ↑ THEN Prices Fall ↓

IF Yields Fall ↓ THEN Prices Rise ↑

This inverse relationship may be understood when one looks at the marketplace as a true auction. Assume an investor purchases a 10-year note with a 6% coupon when yields are at 6%. Thus, the investor pays 100% of the face or par value of the security. Subsequently, rates rise to 7%.

The investor decides to sell the original bond with the 6% yield, but no one will pay par as notes are now quoted at 7%. Now he must sell the bond at a discount to par in order to move the bond. I.e., rising rates are accompanied by declining prices.

Falling rates produce the reverse situation. If rates fall to 5%, our investment yields more than market rates. Now the seller can offer it at a premium to par. Thus, declining rates are accompanied by rising prices.

Should you hold the note until maturity, you would receive the par or face value. In the meantime, of course, one receives semi-annual coupon payments.

Quotation Practices

Unlike money market instruments (including bills and Eurodollars) that are quoted on a yield basis in the cash market, coupon-bearing securities are typically quoted in percent of par to the nearest 1/32nd of 1% of par. For example, one may quote a note at 99-27. This equates to a value of 99% of par plus 27/32nds. The decimal equivalent of this value is 99.84375. Thus, a one million-dollar face value security might be priced at $998,437.50. If the price moves by 1/32nd from 97-27 to 97-28, this equates to a movement of $312.50 (per million-dollar face value).

But often, these securities – particularly those of shorter maturities – are quoted in finer increments than 1/32nd. For example, one may quote the security to the nearest 1/64th. If the value of our note in the example above were to rally from 99-27/32nds by 1/64th, it may be quoted at 99-27+. The trailing "+" may be read as +1/64th.

Quotation Practices

Quote	Means	Decimal Equivalent
99-27	99-27/32nds	99.84375% of par
99-272	99-27/32nds + 1/128th	99.8515625% of par
99-27+	99-27/32nds + 1/64th	99.859375% of par
99-276	99-27/32nds + 3/128ths	99.8671875% of par

Or, you may quote to the nearest 1/128th. If, for example, our bond were to rally from 99-27/32nds by 1/128th, it might be quoted on a cash screen as 99-272. The trailing "2" may be read as +2/8ths of 1/32nd; or, 1/128th. If the security rallies from 99-27/32nds by 3/128ths, it may be quoted as 99-276. The trailing "6" may be read as +6/8ths of 1/32nd or 3/128ths.

The normal commercial "round-lot" in the cash markets is $1 million face value. Anything less is considered an "odd-lot." However, you can purchase Treasuries in units as small as $1,000 face value. Of course, a dealer's inclination to quote competitive prices may dissipate as size diminishes.

Accrued Interest and Settlement Practices

In addition to paying the (negotiated) price of the coupon-bearing security, the buyer also typically compensates the seller for any interest accrued between the last semi-annual coupon payment date and the settlement date of the security.

- It is Friday, July 24, 1998. You purchase $1 million face value of the 5-3/8% security of June 2000 (a two-year note) for a price of 99-27 to yield 5.46%, for settlement on Monday July 27. In addition to the price of the security, you must further compensate the seller for interest accrued between June 30, 1998 (the issue date) and the settlement date of July 27. The total purchase price is $1,002,381.

Price of Note	$ 998,438
Accrued Interest	$ 3,944
Total	$ 1,002,382

Typically, securities are transferred through the Fed wire system from the bank account of the seller to that of the buyer vs. cash payment. That transaction is concluded on the settlement date – which may be different from the transaction date.

It is typical to settle a transaction on the business day subsequent to the actual transaction. Thus, if you purchase the security on a Thursday, you typically settle it on Friday. If purchased on a Friday, settlement generally occurs on the following Monday.

Sometimes, however, a "skip date" settlement is specified. For example, one may purchase a security on Monday for skip date settlement on Wednesday. Or, "skip-skip date" settlement on Thursday; "skip-skip-skip date" settlement on the Friday, etc. Theoretically, there is no effective limitation on the number of days over which one may defer settlement – thus, these cash securities may effectively be traded as a forward.

Treasury Auction Cycle

Treasury bonds, notes and bills are auctioned on a regular schedule by the U.S. Treasury which accepts bids — quoted in yield terms — from securities dealers. A certain amount of each auction is set aside, to be placed on a non-competitive basis at the average yield filled.

Prior to the actual issuance of specific Treasuries, they may be bought or sold on a "WI" or "When Issued" basis. Prior to the actual auction, WI's — bids and offers — are quoted as a yield. Once the security is auctioned and the results are announced, the Treasury will affix a particular coupon to the issue — near prevailing yields. At that time, the security may be quoted on a price rather than a yield basis. Trades previously concluded on a yield basis are settled against a price on the actual issue date of the security, calculated per standard price-yield formulae.

Security dealers purchase these securities and subsequently market them to their customers including pension funds, insurance companies, banks, corporations and retail investors.

The most recently issued securities of a particular maturity are referred to as "on-the-run" securities. On-the-runs are typically the most liquid and actively traded of Treasury securities and, therefore, are often referenced as pricing benchmarks. Less recently issued securities are known to as "off-the-run" securities and tend to be less liquid.

The Treasury currently issues 3-month, 6-month and 1-year bills; 2-year, 5-year and 10-year notes; and 30-year bonds on a regular schedule. In the past, the Treasury had also issued securities with a 3-year, 4-year, 7-year and 20-year maturity.

U.S. Treasury Auction Schedule

	Maturity	Auctioned
Treasury Bills	3 Month	
	& 6 Month	Weekly
	1 Year	*
Treasury Notes	2 Year	Monthly
	3 Year	*
	5 Year	Feb / May / Aug / Nov
	10 Year	Feb / May / Aug / Nov
Treasury Bonds	30 Year	*

* The U.S. Treasury has discontinued the issuance of 1-year bills, 3-year notes and 30-year bonds since this paper was first published.

The 'Run'

If you were to ask a cash dealer for a quotation of "the run," he would quote yields associated with the on-the-run securities from the current on-the-run 3-month bill to the 30-year bond. The most recently issued 30-year bond is sometimes referred to as the "long-bond" because it is the longest maturity Treasury available.

Quoting 'the Run'

(As of Friday, July 24, 1998)

	Coupon	Maturity	Bid	Ask	Chg	Ask Yield
3-Mo. Bill	Na	10/29/98	5.01%	5.00%	+0.08%	5.14%
6-Mo Bill	Na	1/28/99	5.02%	5.01%	-0.01%	5.21%
1-Yr Bill	Na	7/22/99	5.07%	5.06%	-	5.33%
2-Yr Note	5-3/8%	6/00	99-26	99-27	-1	5.46%
3-Yr Note	5-5/8%	5/01	100-13	100-14	-1	5.45%
5-Yr Note	5-3/8%	6/03	99-19	99-20	-2	5.46%
10-Yr Note	5-5/8%	5/08	101-10	101-11	-3	5.45%
30-Yr Bond	6-1/8%	11/27	106-08	106-09	-11	5.68%

Source: Telerate/Cantor Fitzgerald

The long bond is also referred to as the "new bond." Thus, the second most recently issued bond is referred to as the "old bond," the third most recently issued bond is the "old-old bond," the fourth most recently issued bond is the "old-old-old bond." As of this writing, the long bond may be identified as the 6-1/8% bond maturing in November 2027; the old bond is the 6-3/8% bond of August 2027; the old-old bond is the 6-5/8% of February 2027; the old-old-old bond is the 6-1/2% of November 2026.

Beyond that, one is expected to identify the security of interest by coupon and maturity. For example, the "6-3/4s of '26" refers to the bond with a coupon of 6-3/4% maturing on August 15, 2026.

Most Recently Issued Thirty-Year Bonds

(As of Friday, July 24, 1998)

	Coupon	Maturity	Bid	Ask	Chg	Ask Yield
	6-7/8%	8/25	114-27	114-31	-11	5.77%
	6%	2/26	103-05	103-07	-10	5.77%
	6-3/4%	8/26	113-15	113-19	-11	5.77%
Old-Old-Old Bond	6-1/2%	11/26	110-05	110-09	-11	5.76%
Old-Old Bond	6-5/8%	2/27	112-01	112-05	-11	5.75%
Old Bond	6-3/8%	8/27	108-31	109-00	-16	5.73%
Long Bond	6-1/8%	11/27	106-08	106-09	-11	5.68%

Source: Telerate/Cantor Fitzgerald

One important provision is whether or not the bond is subject to call. A "callable" bond is one where the issuer has the option of redeeming the bond at a stated price — usually 100% of par — prior to maturity. If a bond is callable, it may be identified by its coupon, call and maturity date. I.e., the 11-3/4% of November 2009-14 is callable beginning in November 2009 and matures in 2014.

Prior to the February 1986 auction, the U.S. Treasury typically issued 30-year bonds with a 25-year call feature. That practice was discontinued at that time, however, as the Treasury instituted its "Separate Trading of Registered Interest and Principal on Securities" or STRIPS program with respect to all newly issued 10-year notes and 30-year bonds.[1]

Quoting 'the Roll' and the Importance of Liquidity

Clearly, traders who frequently buy and sell are interested in maintaining positions in the most liquid securities possible. As such, they tend to prefer on-the-run as opposed to off-the-run securities.

It is intuitive that on-the-runs will offer superior liquidity when one considers the "life-cycle" of Treasury securities. Treasuries are auctioned — largely to broker-dealers — who subsequently attempt to place the securities with their customers. Often these securities are purchased by investors who may hold the security until maturity. At some point, securities are "put-away" in an investment portfolio until their maturity. Or, they may become the subject of a strip transaction per the STRIPS program.

In any event, as these securities find a home, supplies may become rare — bid/offer spreads inflate and the security becomes somewhat illiquid.

Liquidity is a valuable commodity to many. Thus, you may notice that the price of on-the-runs tends to be bid up — resulting in reduced yields — relative to other similar maturity securities. This tends to be most noticeable with respect to the 30-year bond.

Traders will frequently quote a "roll" transaction where one sells the old security in favor of the new security. The "old bond" in our table above was quoted at a yield of 5.73% while the "new bond" was seen at 5.68%. Clearly, someone is willing to give up 5 basis points (0.05%) in yield for the privilege of holding the new bond. In other words, liquidity is worth about 5 basis points.

Dealers may quote a bid/offer spread in this transaction, offering the opportunity to sell the old bond/buy the new bond; or, buy the old bond/sell the new bond, in a single transaction.

Repo Financing

Leverage is a familiar concept to futures traders. Just as one may margin a futures position and thereby effectively extend one's capital, the Treasury markets likewise permit traders to utilize "repo" financing agreements to leverage Treasury holdings.

A repurchase agreement, repo or simply RP represents a facile method by which one may borrow funds — typically on a very short-term basis — collateralized by Treasury securities. In a repo agreement, the lender will wire transfer same-day funds to the borrower; the borrower wire transfers the Treasury security to the lender — with the provision that the transactions are reversed at term with the lender wiring back the original principal plus interest.

1 The STRIPS program was created to facilitate the trade of zero-coupon Treasury securities. Prior to 1986, a variety of broker dealers including Merrill Lynch and Salomon Bros. issued zero-coupon securities collateralized by Treasuries under acronyms such as TIGeRs and CATS. For example, if you buy a 10-year Treasury, you can create zero coupon securities of a variety of maturities by marketing the component cash flows. By selling a zero collateralized by a coupon payment due in five years, one creates a five-year zero; or, one may create a ten-year zero by selling a zero collateralized by the principal payment. They engaged in this practice because the market valued the components of the security more dearly than the coupon payments and principal payment bundled together. Today, one might notice that the yield on a Treasury STRIP is usually less than a comparable maturity coupon-bearing Treasury. Beginning with 10s and 30s issued in February 1986, the Treasury began assigning separate CUSIP numbers to the principal value and to tranches of coupon payments associated with these securities. A CUSIP number is a code unique to each security and is necessary to wire-transfer and, therefore, market a security. Thus, the Treasury STRIPS market was created. These securities are most popular when rates are high and, therefore, the price of the zero may be quite low.

The borrower is said to have executed a repurchase agreement; the lender is said to have executed a reverse repurchase agreement. Many banks and security dealers will offer this service — once the customer applies and passes a requisite credit check. The key to the transaction, however, is the safety provided the lender by virtue of the receipt of the (highly-marketable) Treasury security.

These repo transactions are typically done on an overnight basis — but may be negotiated for a term of one-week, two-weeks, a month. Overnight repo rates are typically quite low — in the vicinity of Fed Funds.

Any Treasury security may be considered "good" or "general" collateral. Sometimes when particular Treasuries are in short supply, dealers will announce that the security is "on special" and offer below-market financing rates in an effort to attract borrowers.

2. There Are Yields and There Are Yields

Not all yields are created equal. Thus, fixed income traders must be careful to ensure that, when comparing yields, they are comparing similarly constructed figures per similar assumptions.

Discount Yield

If you purchase $1 million face value of a bill, you pay less than $1 million and receive $1 million some days (d) later at term. The difference between the $1 million face value (FV) and the actual price of the bill is referred to as the discount (D). The rate (r) is known as a "discount yield." The price (P) paid to purchase a bill may be calculated as follows.

$$D = FV [r \times (d / 360)]$$

$$P = FV-D = FV – FV [r \times (d / 360)]$$

- Earlier, we quoted a (3-month) T-bill maturing October 29 at a discount yield of 5.00% as of July 24. There were 94 days to maturity. The price of a $1 million face value unit of this bill may be calculated as follows. This implies a discount of $13,056 and a price of $986,944.

$$\text{Discount} = \$1,000,000 [0.05 \times (94 / 360)] = \$13,056$$

$$\text{Price} = \$1,000,000 - \$1,000,000 [0.05 \times (94 / 360)] = \$986,944$$

However, this discount yield of 5.00% applies two incorrect assumptions: (1) There are 360 days in a year when in fact there are 365 days; and (2) The principal value or investment is $1,000,000 when in fact it is less.

Money Market Yields (MMYs)

Money market yields or MMYs should be well understood by traders active in Eurodollar markets. MMYs do not suffer from the mistaken assumption that the investment value is the amount returned upon maturity. This is because Eurodollars are "add-on" instruments where one invests the stated face value and received the original investment plus interest at term. Repo transactions are likewise quoted on a MMY basis.

$$\text{Interest} = FV [r \times (d / 360)]$$

- If one were to purchase a $1 million face value unit of 94-day Eurodollars with a 5.00% (MMY), one would receive the original $1 million investment plus $13,056 at the conclusion of 94 days.

$$\text{Interest} = \$1,000,000 [0.05 \times (94 / 360)] = \$13,056$$

Note that the interest of $13,056 is the same as our T-bill example. Clearly, however, one would prefer to invest only $986,944 to earn $13,056 than to invest $1,000,000. In order to render a discount yield (DY) comparable with a money market yield (MMY), use the following formula.

$$MMY = [(FV / D) – 1] \times (360 / d)$$

- In our example, a discount yield of 5.00% equates to a money market yield of 5.07%.

$$MMY = [(\$1,000,000 / \$986,944) – 1] \times (360 / 94) = 5.07\%$$

Bond Equivalent Yields (BEYs)

MMYs may represent a step up from a discount yield. Yet they still suffer from the mistaken assumption that there are but 360 days in a year (a "money-market" year!).

Accordingly, we must convert those DYs or MMYs in order to render them comparable to a bond or note yield quotation — a bond-equivalent yield (BEY).

$$BEY = MMY \times (365 / 360)$$

We calculated a money market yield of 5.07% in our previous example. Let's convert that figure to a bond-equivalent yield. Note that this figure of 5.14% is quoted in our previous table as the "ask yield" and is comparable to the yields associated with the coupon-bearing securities.

$$5.14\% = 5.07\% \times (365 / 360)$$

Complicating the calculation is the fact that notes and bonds offer semi-annual coupon payments. Thus, money market instruments that require the investor to wait until maturity for any return do not permit interim compounding. This means that the formula provided above is only valid for instruments with less than 6-months (183-days) to term. If there are 183 or more days until term, use the following formula.

$$BEY = \frac{(-d/365) + \sqrt{([d/365]^2 - [(2d/365) - 1]\,[1 - (1/price)])}}{[(d/365)-0.5]}$$

- Above, we had shown a 1-year bill (with 360 days until maturity) quoted at a DDY yield of 5.06%. The price of the bill can be quoted as $949,400:

$$Price = \$1,000,000 - \$1,000,000\,[0.0506 \times (360 / 360)] = \$949,400$$

Substituting into our formula, we arrive at a BEY of 5.33% — which you will note is quoted in our table above as "ask yield" and is comparable to yields quoted on coupon bearing securities.

$$BEY = \frac{(-360/365) + \sqrt{([360/365]2 - [((2 \times 360)/365) - 1]\,[1 - (1/0.9494)])}}{[(360/365)-0.5]} = 5.33\%$$

If you are looking at an "add-on" instrument such as a Eurodollar, substitute (FV plus Interest/1) for the term (1/Price) in our EBY calculation.

3. Measuring Risk of Coupon Bearing Securities

"You can't manage what you can't measure" is an old saying with universal application. In the fixed income markets, it is paramount to assess the volatility of one's holdings in order reasonably to manage them.

Two particular characteristics of a coupon-bearing security will clearly impact upon its volatility: its maturity and its coupon. Defining volatility as the price reaction of the security in response to changes in yield …

The Longer the Maturity ↑ the Greater the Volatility ↑

The Higher the Coupon ↑ the Lower the Volatility ↓

All else held equal, the longer the maturity of a bond, the greater its price reaction to a change in yield. This may be understood when one considers that the implications of yield movements are felt over longer periods, the longer the maturity.

On the other hand, high coupon securities will be less impacted, on a percentage basis, by changing yields than low coupon securities. This may be understood when one considers that high coupon securities return a greater portion of one's original investment sooner than low coupon securities. Your risks are reduced to the extent that you hold the cash!

There are several ways to measure the risks associated with coupon-bearing (and money-market) instruments including basis point value (BPV) and duration.

Measuring Volatility
(As of Friday, July 24, 1998)

	Coupon	Maturity	Ask	Ask yield	BPV	Duration
3-Mo Bill	Na	10/29/98	5.00%	5.14%	$26.11	0.26
6-Mo Bill	Na	1/28/99	5.01%	5.21%	$51.39	0.51
1-Yr Bill	Na	7/22/99	5.06%	5.33%	$100.00	1.00
2-Yr Note	5-3/8%	6/00	99-27	5.46%	$180.46	1.81
3-Yr Note	5-5/8%	5/01	100-14	5.45%	$257.46	2.56
5-Yr Note	5-3/8%	6/03	99-20	5.46%	$426.03	4.28
10-Yr Note	5-5/8%	5/08	101-11	5.45%	$758.21	7.48
30-Yr Bond	6-1/8%	11/27	106-09	5.68%	$1,487.03	13.99

Basis Point Value (BPV)

BPV represents the absolute price change of a security given a one basis point (0.01%) change in yield. These figures may be referenced using any number of commercially available quotation services or software packages. BPV is normally quoted in dollars based on a $1 million (round-lot) unit of cash securities. The following table depicts the BPV of the securities on-the-run as of July 24, 1998.

- If the yield on the 5-year note should rally from 5.46% to 5.47%, this implies a $426.03 decline in the value of the security.

Duration

If BPV measures the absolute change in the value of a security given a yield fluctuation, duration may be thought of as a measure of relative or percentage change. The duration (typically quoted in years) measures the expected percentage change in the value of a security given a one-hundred basis point (1%) change in yield.

Duration is calculated as the average weighted maturity of all the cash flows associated with the bond, i.e., repayment of "corpus" or face value at maturity plus coupon payments — all discounted to their present value.

- The 10-year note has a duration of 7.48 years. This implies that if its yield advances from 5.45% to 6.45%, we expect a 7.48% decline in the value of the note.

In years past, it had been commonplace to evaluate the volatility of coupon-bearing securities simply by reference to maturity. But this is quite misleading. For example, if one simply examines the maturities of the current 2-year note and 10-year note, one might conclude that the 10-year is 5 times as volatile as the 2-year. But by examining durations, we reach a far different conclusion: the 10-year note (duration of 7.48 yrs.) is only about 4.13 times as volatile as the 2-year note (duration of 1.81 yrs.). The availability of cheap computing power has made duration analysis as easy as it is illuminating.

4. The Shape of the Yield Curve

The shape of the yield curve, if closely studied, represents a most revealing topic. In particular, the shape of the yield curve may be interpreted as an indicator of the direction in which the market believes interest rates may fluctuate. Let's look at this topic more closely and find ways to trade on expectations regarding the changing shape of the curve.

Explaining the Shape of the Curve

There are three fundamental theories that address the shape of the yield curve. These three theories are successively more sophisticated yet complementary. These theories are: (1) the expectations hypothesis; (2) the liquidity hypothesis; and (3) the segmentation hypothesis.

Let's begin by assuming that the yield curve is flat, i.e., investors normally express no particular preference for long- vs. short-term securities. Hence, long- and short-term securities have similar yields. The expectations hypothesis alters this assumption with the supposition that fixed income market participants are basically rational. They will alter the composition of their portfolios to correspond to the anticipated direction of interest rates.

For example, portfolio managers will shift their investments from the long-term to the short-term in anticipation of rising interest rates and falling fixed income security prices. This is due to the fact that longer-term securities tend to react more dramatically to shifting rates than do shorter-term securities (as measured by BPV or duration).

By selling long-term securities and buying short-term securities, the portfolio manager assumes a defensive posture. The effects of this strategy are similar to those associated with a portfolio manager who hedges by selling futures against long-term security holdings.

By shortening the average maturity of their holdings in a rising rate environment, investors will tend to bid up the price of short-term securities and drive down the price of long-term securities. Thus, short-term yields fall while long-term yields rise, i.e., the shape of the yield curve steepens.

On the other hand, portfolio managers will tend to lengthen the maturity of their portfolios in anticipation of falling rates by selling short-term securities and buying long-term securities. This activity will have the effect of bidding up the price of long-term securities and drive down the price of short-term securities. Thus, long-term yields will fall while short-term yields rise, i.e., the yield curve flattens or inverts!

Yields Expected to Rise → Yield Curve Steepens

Yields Expected to Fall → Yield Curve Flattens

Thus, the shape of the curve may be used as an indicator of the possible direction in which yields may fluctuate.

The *liquidity hypothesis* rejects the initial proposition described above. According to this theory, investors are not indifferent between long- and short-term securities even when yields are expected to remain stable.

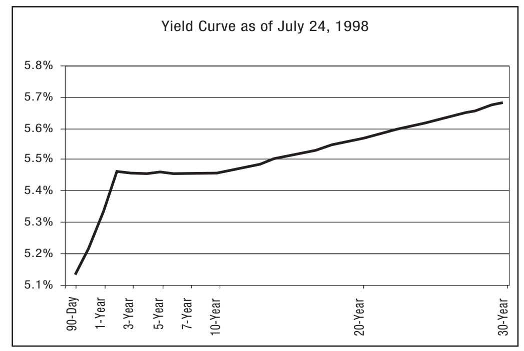
All else being equal, investors will tend to prefer short-term securities over long-term securities. A long-term security implies that an investor commits his funds over a lengthy period of time. Short-term investments, on the other hand, may roll over frequently, providing the investor with augmented flexibility to alter the composition of the portfolio to correspond to breaking events.

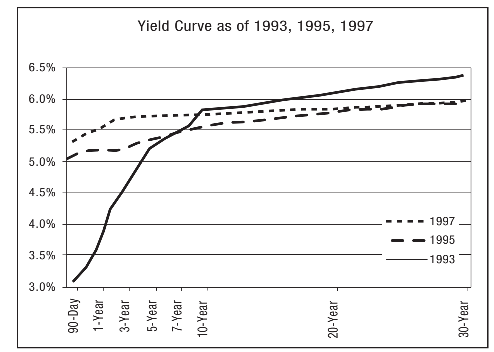
In other words, short-term investments are close to cash. Given similar yields, it is better to own the more liquid and readily divestible security. Thus, long-term securities must pay a "liquidity premium." So as a rule, long-term yields tend to exceed short-term yields even when yields are expected to remain relatively stable. I.e., there is a natural upwards bias to the yield curve.

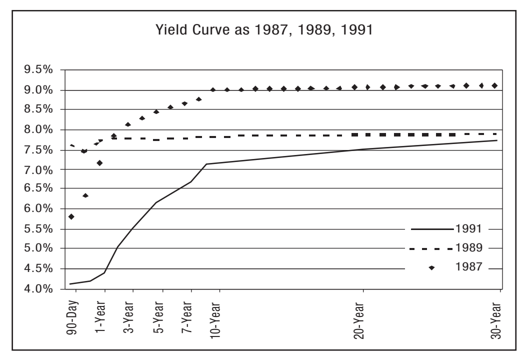
The *segmentation hypothesis* attacks the second proposition associated with the expectation hypothesis. Namely, that investors are ready and willing to alter the structure of their portfolios quickly and efficiently to take advantage of anticipated yield fluctuations. While portfolios may be altered to a limited degree in anticipation of yield curve shifts, investors are often constrained in regard to how quickly and extensively they can adjust their holdings.

For example, regulatory requirements often restrict pension funds, requiring them to be invested in low-risk fixed income investments. The liabilities assumed by insurance companies may prompt them to invest in long-term securities. Other restrictions may be self-imposed. Finally, bid/ask spreads in some segments of the fixed income markets may be prohibitive. As such, "kinks" are often observed in the fixed income markets.

While the liquidity and segmentation hypotheses attack certain aspects of the expectations theory, they are all basically complementary. The latter two theories may be thought of as refinements of the expectations theory.







Normal vs. Inverted Yield Curve

Typically, we expect the yield curve to be somewhat upwardly sloped with long-term yields exceeding short-term yields. A strong upward slope may be interpreted as an indication that the market believes that yields will rise (and prices decline). A flat, negatively sloped or inverted yield curve may be interpreted as an indication that the market believes that yields will decline (and prices rise).

Since the early 1980s, yields in the United States have generally been on the decline. As a result, the yield curve has typically been upwardly sloped.

One would have to go all the way back to the early 1980s to find a dramatically inverted yield curve environment. Recall that in the October 1979, Fed Chairman Volcker initiated a campaign to control inflation (in the long-term) by limiting money supply growth at the risk of allowing rates to rise sharply (in the short-term). As a result, short-term yields peaked near 20% while long-term yields peaked near 14% — a dramatically inverted yield curve. One might argue that this foretold of eventually declining yields. Rates did in fact decline as inflation was reigned in from the double-digit levels witnessed in the late 70s.

While the yield curve is relatively flat as of this writing, we would have to go back to the Spring of 1989 to find a situation where the yield curve was slightly inverted – or at least quite flat.

Yield Spreads

It is quite common to speculate on the spread — quoted in yield — between securities of various maturities. In order to make such a quotation, of course, one must ascertain that you are "comparing apples with apples" — or using BEYs across the board! The following table depicts yield spreads in on-the-run securities.

Yield Curve Spreads
(As of Friday, July 24, 1998)

	Yield	3-mo	6-mo	1-yr	2-yr	3-yr	5-yr	10-yr	30-yr
3-Mo	5.14%	–							
6-Mo	5.21%	0.07%	–						
1-Yr	5.33%	0.19%	0.12%	–					
2-Yr	5.46%	0.32%	0.25%	0.13%	–				
3-Yr	5.45%	0.31%	0.24%	0.12%	-0.01%	–			
5-Yr	5.46%	0.32%	0.25%	0.13%	0.00%	0.01%	–		
10-Yr	5.45%	0.31%	0.24%	0.12%	-0.01%	0.00%	-0.01%	–	
30-Yr	5.68%	0.54%	0.47%	0.35%	0.22%	0.23%	0.22%	0.23%	–

If you believe that the yield spread will widen while the curve steepens, i.e., the yield on the long-term security will rise relative to the yield on the short-term security, you might "buy the curve." You do so by buying the short-term security and selling the long-term security. If you believe the yield spread will narrow while the curve flattens or inverts, i.e., the yield on the long-term security will decline relative to the yield on the short-term security, you "sell the curve." This is accomplished by selling the short-term security and buying the long-term security.

Buy the Curve → Buy ST / Sell LT

Sell the Curve → Sell ST / Buy LT

But consider what might happen if you were to sell the curve (in anticipation of falling yields and a flattening curve) by selling an equal face value of short-term and long-term securities. Long-term securities are more reactive to changing yields than short-term securities. Thus, it is possible that yields fall and the curve flattens but you lose money! In other words, your prediction is correct but you still lose money — because you failed to recognize that securities of varying coupons and maturities react differently to fluctuating yields.

Fortunately, the BPVs or durations discussed above provide a convenient reference by which to "weight" the spread. For example, if one were to sell 2s/10s (sell the 2-year note/buy the 10-year note), one might weight the spread on a ratio of 4.13 to 1. In other words, sell 4.13 face value units of the two-year to one face value unit of the 10-year. Given that a commercial round-lot is generally considered to be $1 million face value, one might sell $4 million 2-years/buy $1 million 10-years; or, sell $33 million 2-years/buy $8 million 10-years.

Yield Curve Spread Weights
(As of Friday, July 24, 1998)

	Duration	3-mo	6-mo	1-yr	2-yr	3-yr	5-yr	10-yr	30-yr
3-Mo	0.26	–							
6-Mo	0.51	1.96	–						
1-Yr	1.00	3.85	1.96	–					
2-Yr	1.81	6.96	3.55	1.81	–				
3-Yr	2.56	9.85	5.02	2.56	1.41	–			
5-Yr	4.28	16.46	8.39	4.28	2.36	1.67	–		
10-Yr	7.48	28.77	14.66	7.48	4.13	2.92	1.75	–	
30-Yr	13.99	53.81	27.43	13.99	7.73	5.46	3.27	1.87	–

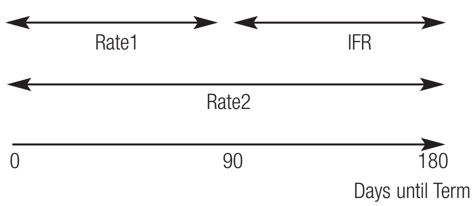
By weighting the spread in this way, you assure that the profit or loss on your spread will be a function of the changing shape of the curve — not yield levels *per se*.

5. The Yield Curve, Implied Forwards and Strips

There is much valuable information implicit in the shape of the yield curve, not the least of which is an indication of where short-term interest rate futures prices should be. Let us study how one might empirically derive such information.

Implied Forward Rates

An "implied forward rate" or IFR answers this question: what short-term yield may be expected to prevail in the future? For example, what yield will be associated with a 90-day investment instrument 90 days in the future?



The 90-day implied forward rate 90 days in the futures IFR(90,90) may be found as a function of the 90-day term rate Rate1 or simply "R1" and the 180-day term rate Rate2 or simply "R2". (180 days = 90 +90.). Let's designate the number of days in each period as $days_1$ or simply d_1=90 days; $days_2$ or simply d_2=180 days; and $days_3$ or simply d_3=90 days.[2]

The assumption is that an investor should be indifferent between investing for a 6-month term and investing at a 3-month term, rolling the proceeds over into a 3-month investment 90 days hence.

$$1+ R_2 (d_2/360) = [1+R_1(d_1/360)]\ [1+IFR(d_3,d_1)(d_3/360)]$$

Solving the equation for IFR:

$$1+IFR (d_3,d_1) (d_3/360) = [1+R_2 (d_2/360)] / [1+R_1 (d_1/360)]$$

$$IFR (d_3,d_1) (d_3/360) = [[1+R_2 (d_2/360)] - 1] / [1+R_1(d_1/360)]$$

$$IFR (d_3,d_1) = [[1+R_2 (d_2/360)] / [(d_3/360)[1+R_1 (d_1/360)]]] - [1 / (d_3/360)]$$

- Assume that the 90-day term rate equals 5-11/16% or 5.6875% while the 180-day term rate equals 5-3/4% or 5.75%. What is the 90-day implied forward rate 90 days hence?

$$IFR_{90,90} = [[1+(0.0575)(180 / 360)] / [(90/360) [1+(0.056875)(90/360)]]] - [1 / (90/360)] = 0.05731 \text{ or } 5.731$$

This analysis represents a precise but somewhat less than intuitive method for determining the IFR. Let's explore a more intuitive means of determining IFRs given the conditions described above.

- If one invests one million dollars at a rate of 5.75% over a 180-day or six-month period, one would have accumulated the total of $28,750 as follows:

$1,000,000 x 0.0575 x (180 / 360) = $28,750

As an alternative, one might have invested that one million dollars at the spot 90-day rate of 5.6875% and earned $14,219 over the first 90-day period. The IFR is the rate of return one must earn on the second 90-day period in order to become indifferent between a term 180-day investment and the prospect of investing for 90 days, subsequently rolling over into another 90-day investment 90 days hence at the IFR. Clearly, one must earn a total of $14,531 over the second 90-day period in order to accumulate the sum total of $28,750.

$1,000,000 x 0.056875 x (90/360)	= $14,219
$1,014,219 x IFR x (90/360)	= $14,531
	= $28,750

Note that the second strategy means that you will benefit from the compounding effect. By investing in a 90-day term instrument, one realizes the return of the original one million dollars plus interest of $14,219 at the conclusion of 90 days. Thus, one has more cash to invest during the second 90-day period than the first. Solving for the IFR:

2 Note that, for the sake of convenience, we assume that there are precisely 90 days in each period from Eurodollar futures maturity date to date. Clearly, this is not the case in practice where this assumption introduces gaps between the value date of one futures contract + 90 days and the value date of the next futures contract. We further assume that the "stub" – or the period between the current (settlement) date and the value date for the first futures contract – is also precisely 90 days.

IFR = $14,531 / [$1,014,219 x (90/360)]
= 5.731%

Note that this IFR is slightly above the spot 90-day rate. This implies an expectation of slightly rising yields in the future.

- The 90-day rate is 6.00% and the 180-day rate is 6.25%. I.e., the yield curve displays a "normal" upward slope. find the 90-day IFR for 90 days hence.

$$IFR_{90,90} = [[1+(0.0625)(180/360)] / [(90/360)\ [1+(0.06)\ (90/360)]]] - [1 / (90/360)]$$

= 0.06404 or 6.404%

This IFR exceeds the 90-day and the 180-day term rates. Thus, it implies an expectation of rising yields and is consistent with the expectations hypothesis discussed above.

* The 90-day rate is 6.00% and the 180-day rate is 5.75% – well below the shorter-term rate. Thus, the yield curve is negatively sloped or inverted. find the 90-day IFR for 90 days hence.

$$IFR_{90,90} = [[1+(0.0575)(180/360)] / [(90/360)\ [1+(0.06)(90/360)]]] – [1 / (90/360)]$$

= 0.05419 or 5.419%

This IFR is lower than either the 90-day or the 180-day term rates. This implies an expectation of falling yields and is consistent with the expectations hypothesis discussed above.

* The 90-day rate is 6.00% and the 180-day rate is 6.00%. Thus, the yield curve is flat. find the 90-day IFR for 90 days hence.

$$IFR_{90,90} = [[1+(0.06)(180/360)] / [(90/360)[1+(0.06)(90/360)]]] – [1 / (90/360)]$$

= 0.05911 or 5.911%

This IFR is lower than the 90-day and the 180-day term rates. This implies an expectation of falling yields! This would appear superficially to run contrary to the expectations hypothesis as one might assume that a flat curve is indicative of stable expectations. We can explain this discrepancy from either a mathematical or an intuitive viewpoint.

	90-day Rate	180-day Rate	IFR
Curve Steepens	6.00%	6.25%	6.404%
Curve Inverts	6.00%	5.75%	5.419%
Curve Flat	6.00%	6.00%	5.911%

Mathematically, it is clear that a flat curve gives rise to falling IFRs. This is due to the effect of compound interest. Note that over the second 90-day period, the investor has more cash to reinvest. The 180-day investment means that the investor's cash is tied up for the full 180-day term with no opportunity to benefit from the effects of compound interest. Thus, the investor may be content with a slightly lower rate over the second 90-day period.

From an intuitive standpoint, the liquidity hypothesis suggests that, in the absence of expectations of rising or falling rates, the yield curve should maintain a slight upward slope. This slight upward or "normal" slope is expected given investors' normal preference for shorter-term, more liquid, as opposed to longer-term and presumably less liquid securities. Thus, a flat curve is actually indicative of an expectation for slightly falling interest rates.

IFRs and Futures

Whereas the cost of carry analysis discussed in prior sections is often quite useful in identifying "normal" relationships in many futures markets, the IFR concept may be referenced as an indication of where short-term interest rate futures may be trading.

A Eurodollar or T-bill futures contract essentially represents a three-month or 90-day term investment taken some n days in the future. This is readily comparable to an IFR. In fact, by comparing IFRs to futures prices, you may identify arbitrage opportunities to the extent that IFRs basically represent where these short-term futures should be trading!

- Assume that it is June. Consider the following hypothetical interest rate structure in the Eurodollar (ED) futures and cash markets:

September ED futures	94.32 (5.68%)
December ED futures	94.285 (5.715%)
March ED futures	94.345 (5.655%)
3-Month investment	offer @ 5.6875%
6-Month investment	offer @ 5.75%
9-Month investment	offer @ 5.78125%

Which is the better investment: (1) Invest in Eurodollars maturing in six months hence yielding 5.75%; (2) Invest for the next three months at 5.6875% and buy September futures at 94.285; or (3) Invest in the 9-month spot at 5.78125% and sell March futures at 94.345 (5.655%)?

Alternative #2 presumes that you will invest for the first three months at 5.6875%, and for the next three month period (at a price determined or "locked-in" by virtue of the fact that you are long September futures) at 5.715%. The total return may be found by using the IFR equation as follows:

$$1+R\,(0.5) = [1+0.056875(0.25)][1+0.05715(0.25)]$$

$$R = [[1+0.056875(0.25)][1+0.05715(0.25)] - 1] / 0.5$$

$$= 5.742\%$$

Alternative #3 means that you invest for nine months at 5.78125% over the next 270 days. But by selling March futures, you effectively commit to selling that Eurodollar investment 180 days hence when it has only 90 days to term. The return may also be found by using the IFR equation as follows:

$$[1+R(0.5)][1+0.05655(0.25)] = [1+0.0578125(0.75)]$$

$$[1+R\,(0.5)] = [1+0.0578125(0.75)] / [1+0.05655(0.25)]$$

$$R = [[(1+0.0578125(0.75)) / (1+0.05655(0.25))] - 1] / 0.5$$

$$= 5.763\%$$

Thus, the third alternative provides a (marginally) superior return at 5.763% relative to the first alternative with a return equal to 5.75% and the second alternative with a return equal to 5.742%.

Short-term futures are driven into line with these implied forward rates because of the availability of arbitrage opportunities. In the foregoing example, an arbitrage might have been constructed by selling the 6-month cash Eurodollar foregoing a return equal to 5.75%; buying the 9-month cash Eurodollar and selling the June futures contract, earning a return equal to 5.76%. Of course it may not be worth the effort for a one basis point return. In any event, the execution of such transactions has the effect of driving the market into equilibrium such that arbitrage opportunities are unavailable.

Strip Transactions

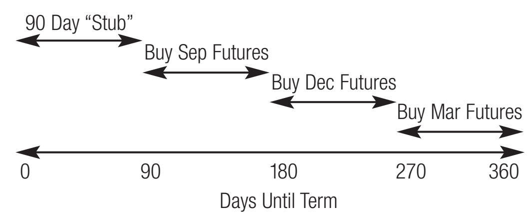
This analysis also brings to the fore the concept of a "strip." A strip may be purchased or sold by buying or selling a series of futures maturing in successively deferred months, usually in combination with a current position in the cash market.

- *A strip* involves the purchase or sale of short-term interest rate futures in successively deferred months, usually in combination with the purchase or sale of the current cash short-term instrument.

In our example, we illustrated the first two components of a strip when we discussed the purchase of the current 3-month ED and purchase of futures calling (nominally) for delivery of 3-month EDs three months hence.

Assume it is June and there are three months until maturity of the nearby (September) Eurodollar futures contract. A strip, albeit a short strip, may be constructed by investing in a 90-day or 3-month Eurodollar instrument (the "stub"), buying September, December and March futures.

When the spot Eurodollar investment matures in 90 days, one (effectively) invests in another 90-day Eurodollar investment by virtue of the fact that one may be long the nearby September contract. Subsequently, when that 3-month investment matures, one invests for another three months at a predetermined or "locked-in" price by virtue of a long position in December futures. When that 3-month investment matures, one invests for yet another three months at a predetermined or "locked-in" price by virtue of a long position in March futures.



The value of this strip may be found with another variation of the IFR formula. This value may be compared against the prospect of simply investing at the spot 12-month rate.

$$\text{Strip} = (360/d) \times [[[1+r_1(d_1/360)][1+r_2(d_2/360)] \ldots [1+r_n(d_n/360)]] - 1\,]$$

- Let's construct an example whereby we can apply our formula to calculate the value of a strip. Consider the following rate structure. Calculate the value of a strip constructed by investing the sum of $100 million in the 3-month "stub," buying September, December and March futures.

3-Month 'Stub'	5.6875%
Sep. ED futures	94.32 (5.68%)
Dec. ED futures	94.285 (5.715%)
Mar. ED futures	94.345 (5.655%)

$$\text{Strip} = (360/360) \times [[[1+0.056875(90/360)][1+0.0568(90/360)][1+0.05715(90/360)]\ [1+0.05655(90/360)]] - 1\,]$$

$$= 5.807\%$$

While this calculation certainly allows one to arrive at the correct solution, it not entirely intuitive. In order to explicitly illustrate how this figure was determined, let us consider the cash flows associated with this series of transactions.

$100,000,000	x	0.056875	x	(90 / 360)	= $1,421,875
$101,421,875	x	0.0568	x	(90 / 360)	= $1,440,191
$102,862,066	x	0.05715	x	(90 / 360)	= $1,469,642
$104,331,708	x	0.05655	x	(90 / 360)	= $1,474,990
					= $5,806,698

Return on Strip = 5.807%

As illustrated above, if one invests $100,000,000 for the first 90-day period at a rate of 5.6875%, one earns $1,421,875. By reinvesting the original $1,000,000 plus the accrued interest over the second 90-day period at a rate of 5.68%, one earns an additional $1,440,191. An additional $1,469,642 plus $1,474,990 are earned over the third and fourth periods for a total of $5,806,698. Thus, one earns $5,806,698 on a $100 million investment over the course of one year or 5.807%.

The value of a strip may be compared to the value of a comparable term investment. In the above example, we would expect a one-year term spot Eurodollar investment to yield 5.81%.

If the term investment were to yield more than 5.807%, buy spot and sell the strip. Should the term investment yield less than 5.807%, sell spot and buy the strip. Arbitrageurs who pursue such trading opportunities will tend to enforce cost of carry pricing through their activities.

Weighting the Strip

Note that in the previous example, the strip buyer becomes the beneficiary of compound interest. In other words, one has more to re-invest over subsequent periods than the amount with which one began. This implies that one should "weight" a strip by purchasing additional quantities of ED futures in subsequent months to account for the effect of compounding.

Assume that you began with an original investment of $100 million. Rather than buying 100 September, 100 December and 100 March futures, you might consider purchasing 101 September, 103 December and 104 March futures – by reference to the compound interest.

Many strips are constructed on a simple unweighted basis — often with the use of packs or bundles. While it may technically be advisable to construct a weighting as above; as a practical matter, one might expect quicker, superior execution with use of packs or bundles.[3]

6. Trading the T-Note/Eurodollar Spread

As discussed above, one might compare the yield on a strip to the yield on a comparable maturity spot Eurodollar investment. A profitable trade might be executed if there are sizable differences in yield.

However, Eurodollars are money market instruments – by definition, a money market instrument is one which has a maturity of one year or less. It is possible to trade Eurodollar strips vs. longer maturity securities such as Treasury notes. We will refer to this Treasury/Eurodollar strip trade as the "T-Note/ED spread." (It is sometimes referred to in the marketplace as a "Term TED spread.")

In its simplest form, one might buy the spread (buy T-Notes/sell Euro strips) in anticipation of a widening yield spread between Treasuries and Eurodollars. Or, one may sell the spread (sell T-Notes/Buy ED strips) in anticipation of a narrowing yield spread between Treasuries and Eurodollars. Thus, this may be considered a credit risk or a "flight to quality" play.

Long the T-Note/ED Spread → Buy T-Notes/Sell ED Strips

Short the T-Note/ED Spread → Sell T-Notes/Buy ED Strips

Spread vs. Arbitrage

Consider the possibility of trading a two-year Eurodollar strip vs. a two-year Treasury note. The effective term associated with either investment may be identical. But this spread is qualitatively different than trading a Eurodollar strip vs. a spot Eurodollar investment.

3 See "Picking Your Spots on the ED Strip: A Graphical Guide to 'Quick and Dirty' Hedging for Treasury Notes" by Frederick Sturm, Fuji Securities, CME Open Interests for a discussion of the relative effectiveness of bundles and packs vs. the more elaborate approach.

In particular, Eurodollars represent private credit risks vs. the reduced public credit risk implied in Treasury yields. Because credit risk is an important issue, this is indeed a "spread" and should not be considered an "arbitrage." This spread is analogous to the "simple" TED spread between Treasury bill and Eurodollar futures.[4]

A Two-Year Strip

In order to assess the value of this spread, it is necessary to "compare apples with apples." In other words, we must ascertain that the yield on the strip compares to the bond equivalent yield associated with the note (BEYnote). In order to illustrate this process, let's consider the construction of a two-year Treasury/ED strip spread. There are alternative ways of quoting this spread but this is the simplest method.[5]

In order to compare the strip to the yield on a two-year note, we find its BEY as follows: (1) find the forward value (FV) of the strip; and (2) use that information to derive a BEY for the strip (BEYstrip).

FV represents the growth in value of a strip over term. The reciprocal of the FV, or "discount factor" DF = 1 / FV, is analogous to the value of a zero-coupon bond. For example, if a strip grows in value from $1.00 to $1.10 over some term (FV = 1.10), its discount factor is 0.9090. I.e., a zero coupon bond priced at 90.90% of par.

Next, we convert that FV into a BEY using the following formula where CP represents the number of semi-annual coupon periods from settlement to maturity of the strip. This figure must include fractional coupon periods.

$$BEY = [FV^{(1/CP)} - 1] \times 2$$

- It is July 24, 1998. Let us quote the spread by comparing a two-year strip vs. the on-the-run two-year Treasury note. The 5-3/8% note of June 2000 is quoted at 99-27 to yield 5.46%.

Futures Value Date	Rate	Days	Forward Value
7/27/98	5.6875%		1.00000
9/16/98	5.680%	51	1.00806
12/16/98	5.715%	91	1.02253
3/17/99	5.655%	91	1.03730
6/16/99	5.670%	91	1.05213
9/15/99	5.700%	91	1.06721
12/15/99	5.810%	91	1.08259
3/15/00	5.750%	91	1.09849
6/21/00	5.780%	98	1.11568
9/20/00	5.800%	91	1.13198

There are CP = 3.85 coupon periods between the value date of July 27, 1998 and the maturity of the two-year note on June 30, 2000. This represents WC = 3 whole coupon payments received in June 1999, December 1999 and June 2000; plus, the fractional period between July 27 and the first coupon payment on December 31. There are D_1 = 157 days from July 27 and December 31. There are D_2 = 184 days between the original issue date of June 30 and December 31.

$$CP = WC + D_1 / D_2$$

$$= 3 + 157 / 184$$

$$= 3.85$$

4 Per a "simple" TED spread, one buys (sells) Treasury bill futures and sells (buys) Eurodollar futures. The spread is quoted as the T-bill futures price less the Eurodollar futures price. Because bills can be expected to offer lower yields than Eurodollars of a comparable maturity due to credit considerations, the spread is positive. One buys the TED (buys bills/sells ED futures) if one expects credit considerations to heat up (a "flight to quality"). Or, one might sell the TED (sell bills/buy ED futures) if one expects credit considerations to become less significant.

5 Other methods of quoting the T-Note/ED futures spread include comparing the "implied Eurodollar yield vs. Treasury yield" or the "fixed basis point spread to Eurodollar futures rates." There are advantages and disadvantages to each approach. See "Measuring and Trading Term TED Spreads" by Galen Burghardt, Bill Hoskins, Susan Kirshner, *Dean Witter Institutional Futures Research Note*, July 26, 1995.

A seemingly minor but important point: our two-year note matures on June 30, 2000 while our strip extends either to June 21, 2000 or September 20, 2000. Thus, we must extrapolate between the forward value $FV_{6/21}$ = 1.11568 and the $FV_{9/20}$ = 1.13198 to find the $FV_{6/30}$ appropriate to the June 30 maturity. There are D1 = 9 days between June 21 and June 30. There are D2 = 91 days between June 21 and September 20. Thus, our extrapolated FV becomes 1.11715.

$$FV_{6/30} = FV_{6/21} + [(D_1 / D_2) \times (FV_{9/20} - FV_{6/21})]$$

$$= 1.11568 + [(9 / 91) \times (1.13198 - 1.11568)]$$

$$= 1.11729$$

Putting it all together, we find a BEY = 5.845%.

$$BEY = [FV^{(1/CP)} - 1] \times 2$$

$$= [1.11729^{(1/3.85)} - 1] \times 2$$

$$= 5.845\%$$

There are additional subtleties that we might have considered. Still, by comparing the yield on our strip at 5.845% to the yield on the note of 5.46%, we have a quote of 38.5 basis points.

$$\text{T-Note/Eurodollar Futures Quote} = BEY_{strip} - BEY_{note}$$

= 5.845% - 5.46%
= 0.385% or 38.5 basis points

Hedge Ratios

The simplest and most straightforward method of identifying a hedge ratio (HR) is simply to compare the basis point value (BPV) of the Treasury note against that of the Eurodollar futures contract.

$$HR = BPV_{note} / BPV_{strip}$$

- Our two-year note has a BPV_{note} = $180.46 per million; Eurodollar futures have a BPV = $25 per million. Our strip, extending some 703 days between July 27, 1998 and June 30, 2000, runs 1.95 years into the future [703/(2 x 360)] — thus, its BPV_{strip} = $195.[6] The hedge ratio may be crudely calculated as 0.925. This suggests the purchase (sale) of roughly 9 strips vs. $10 million face value of the note.

HR = $180.46 / $195 = 0.925

Please note, however, that this is a rough calculation and does not account for many factors. It is clear, for example, the performance of a strip is more heavily affected by changing values in its short-dated as opposed to long-dated components — given the compounding effects discussed above. Further, this HR is based on the unlikely assumption that Eurodollar and Treasury yields will move in parallel.[7]

Non-Parallel Shifts in the Yield Curve

What if yields along the curve do not move up and down in parallel — but rather the shape of the curve is in flux? A flattening or steepening of the curve can have a dramatic effect upon our spread.

One might buy the T-note/sell the strip — in anticipation of a widening yield spread. But this further implies an interest in seeing the yield curve flatten or invert. Or, one might sell the note/buy the strip — in anticipation of a narrowing yield spread. This implies an interest in seeing the yield curve steepen.

Long the T-Note/ED Spread → Benefits from Flattening Curve

Short the T-Note/ED Spread → Benefits from Steepening Curve

6 The BPV of a non-coupon bearing security equals BPV = 0.01% x [FV x (d/360)]. Thus, a $1 million, 90-day Euro futures contract has a BPV = $25 = 0.01% x [$1 million x (90/360)].

7 It is beyond the ambition of this paper to provide painstakingly precise formulae. See "Measuring and Trading Term TED Spreads" by Galen Burghardt, Bill Hoskins, Susan Kirshner, *Dean Witter Institutional Futures Research Note*, July 26, 1995, for a more detailed discussion of these issues.

Financing Treasury Positions: Using Repurchase and Reverse Repurchase Agreements

by Peter Barker

What Is a Repo?

In a basic repo transaction one party sells securities to another party while simultaneously agreeing to buy them back at a set date in the future. The seller of the securities is borrowing money for the term of the repo deal. The interest the borrower pays is expressed as the difference between the selling price and the (higher) repurchase price. It can be looked upon as a collateralized loan.

Example #1: Buy a Treasury note and finance the position for one day

Instrument	5-year T-note 6 5/8% of 3/31/02
Trade Date	April 2, 1997
Settlement Date	April 3, 1997
Note Price	99-19 + ($996,093.75/million)
Accrued Interest (3 days)	$543.03 (3/183 x .033125 x $1,000,000)
Full Price	$996,636.78

The trader buys a 5-year note on April 2 that has to be paid for on April 3, the following day. Because the trader does not have $996,636.78 available in his account to pay for the note, he finances the position by using the note he just purchased as collateral. How does this work?

Trade Date April 2 (Wednesday)

Buy $1,000,000 (Face Value)	6 5/8% of 3/31/02
Full Price	996,636.78

Overnight Repo: Agree to sell the note and receive cash on April 3, and buy the note back on April 4. Selling the note on April 3 is equivalent to borrowing money; buying the note back the next day is equivalent to repaying the loan. The difference between the selling and buying price is the overnight interest rate charge (repo rate at 5.5%).

Sell price:	$996,636.78 (receive cash 4/3)
Buy price:	$996,789.04 (to be paid 4/4)
	($152.26) overnight interest

April 3 (Thursday)

$1,000,000 (face value) 5-year T-note is delivered to the buyer's clearing firm sub-account. The note is then transferred (repo'd out) to a repo counterparty in return for cash. The T-note buyer's clearing firm wires the full price amount of $996,636.78 (proceeds from the sell leg of the repo transaction) to the seller's bank. The original seller has now been paid. The note buyer will pay back the $996,636.78 plus interest on April 4.

Let's assume that the trader sells out of the long position this same day (Thursday), and that the price of the note has not changed.

Trade Date	April 3, 1997
Settlement Date	April 4, 1997
Note Price	99-19+ ($996,093.75/million)
Accrued Interest (4 days)	$724.04 (4/183 x .033125 x $1,000,000)
Full Price	$996,817.79

April 4 (Friday) Settlement Day

The trader receives the $996,817.79 from the previous days' sale. He repays the $996,789.04 overnight repo loan, and receives the note that was loaned out. The note is then sent out to the buyer, so the Treasury note position is flat. Because the price of the note did not change over the day covered in the example (April 2 to April 3), the profit of the trade is simply equal to the difference between the accrued coupon interest received for one day ($181.01), and the repo interest paid out for one day ($152.26), or $28.75.

Example #2: Buy a Treasury note / Finance the position for one day/Sell the Eurodollar strip
Assume the same trade dates and processes shown in Example #1 for the 5-year note, but add the elements of (1) price volatility in the note and (2) a 5-year strip of Eurodollar futures sold against the note as a hedge.

Trade Date April 2 (Wednesday)

Trade Date	April 2, 1997
Settlement Date	April 3, 1997
Note Price	99-19+ ($996,093.75/million)
Accrued Interest (3 days)	$543.03 (3/183 x .033125 x $1,000,000)
Full Price	$996,636.78
Settlement Price	100-00

Assume that this trade is executed the morning of April 2, the market rallies during the day, and the note is marked at 100-00 at 2:00 p.m. The buyer now has a bond worth $1,000,543.03 (clean price plus accrued interest). The trader can obtain use of the cash value of the position by waiting until 2:00 p.m. (Chicago time) to enter into a repurchase agreement.

By doing so, the trader borrows $1,000,543.03 for April 3 (settlement day), and agrees to pay back the original amount plus $152.86 in interest the following day ($1,000,695.89). On settlement day, the trader will be obligated to send the seller only the $996,636.78 purchase price. The trader's Treasury account at day's end on April 2 will show a dollar surplus of $3,906.25, equal to $1,000,543.03 (repo proceeds) minus $996,636.78 (note payment obligation). The cash flows for both legs take place on April 3.

The market rally has a positive effect on the trader's Treasury note position, but a negative effect on his Eurodollar strip position. Assume the rally results in a loss on the short Eurodollar position of $3,500. The settlement variation payment for the Eurodollar side of the trade must be made on April 3, the morning following the trade.

April 3 (Thursday)
The trader receives the 5-year note, and the seller is wired the $996,636.78 purchase price. The $3,906.25 excess is used to cover the $3,500 settlement variation pay amount on the short Eurodollar position due Thursday morning. The trader's net position at this point shows a net excess of $406.25.

Let's assume that the trader sells out of the long note position, and buys back the Eurodollar futures this same day. Assume that the sale price of the note remains 100-00, and Eurodollar prices have not changed since the previous day's close (April 2).

Trade Date	April 3, 1997	
Settlement Date	April 4, 1997	
Bond Price	100-00	($1,000,000)
Accrued Interest (4 days)	$724.04 (4/183 x .033125 x $1,000,000)	
Full Price	$1,000,724.04	

April 4 (Friday) Settlement Day
The trader receives the $1,000,724.04 from the previous day's sale. He repays the $1,000,695.89 overnight repo loan, and receives the note that was loaned out. The note is then sent out to the buyer, so the Treasury position is flat.

The net cash flow on April 3 amounted to $406.25 ($3,906.25 – $3,500). Add one day's interest, say 5.5% (annualized), and this grows to $406.31 by April 4. Add to this the difference between the amount received from the bond sale ($1,000,724.04), and the sum paid on the repurchase agreement ($1,000,695.89), and the result is an additional inflow of $28.15. The net balance as of the end of April 4 is $434.46.

THE KEY POINT: The appreciation of the Treasury note was used to throw off cash to pay the settlement variation on the short Eurodollar strip.

Financing Short Treasury Positions Using Reverse Repurchase Agreements (Reverse REPOs)

What Is a Reverse REPO?
A reverse repo is the opposite side of a repo transaction. In a basic reverse repo transaction one party buys securities from another party while simultaneously agreeing to sell them back at a set date in the future. The buyer of the securities is lending money for the term of the reverse repo deal. The interest the lender earns is expressed as the difference between the buying price and the (higher) selling price. Like repos, reverses can be looked upon as collateralized loans.

Example #3: Sell a Treasury note and "reverse it in" overnight

Instrument	5-year T-note	6 5/8% of 3/31/02
Trade Date	April 2, 1997	
Settlement Date	April 3, 1997	
Note Price	99-19+	($996,093.75/million)
Accrued Interest (3 days)	$543.03 (3/183 x .033125 x $1,000,000)	
Full Price	$996,636.78	

The trader has sold short a 5-year note on April 2 that must be delivered the following day. Because the trader does not already own the note, he must borrow it, then deliver it on settlement day. How does this work?

Trade date April 2 (Wednesday)

Sell $1,000,000 (Face Value)	6 5/8% of 3/31/02
Full Price	$996,636.78

Overnight Reverse: Agree to buy the note and pay cash on April 3, and sell the note back on April 4. Buying the note on April 3 is equivalent to lending money; selling the note back on the following day is equivalent to having the loan repaid. The difference between the buying and selling price is the overnight interest rate earned (repo rate at 5.5%).

Buy price:	$996,636.78 (to be paid 4/3)
Sell price:	$996,789.04 (to be received 4/4)
	$152.26 overnight interest

April 3 (Thursday)
The 5-year T-note is wired (reversed in) to the seller's clearing firm sub-account in return for cash. The note is immediately moved out of the seller's account, and delivered to the buyer's custodial account. The T-note buyer wires the full price amount of $996,636.78 to the seller's clearing firm sub-account. The original buyer has now been satisfied. The note seller will sell back the note on April 4 for $996,636.78 plus interest to the reverse counterparty.

Let's assume that the trader buys back the short position this same day (Thursday), and that the price of the note has not changed.

Trade Date	April 3, 1997
Settlement Date	April 4, 1997
Note Price	99-19+ ($996,093.75/million)
Accrued Interest (4 days)	$724.04 (4/183 x .033125 x $1,000,000)
Full Price	$996,817.79

April 4 (Friday) Settlement Day
The trader pays $996,817.79 for the previous day's purchase, and receives the 5-year note. This note purchased on April 3 is then sent to the reverse repo counterparty in return for $996,789.04 to satisfy the sale leg of the overnight reverse transaction. The Treasury position is now flat. Because the price of the note did not change over the day covered in the example (April 2 to April 3), the loss on the trade is simply equal to the difference between one day's accrued coupon interest ($181.01), and the reverse repo interest received for one day ($152.26), or $28.75.

Example #4: Sell a Treasury note / "Reverse it in" overnight
Buy the Eurodollar strip

Assume the same trade dates and processes shown in Example #3 for the 5-year note, but add the elements of (1) price volatility in the note and (2) a 5-year strip of Eurodollar futures bought against the note as a hedge.

Trade Date April 2 (Wednesday)

Trade Date	April 2, 1997	
Settlement Date	April 3, 1997	
Note Price	99-19+	($996,093.75/million)
Accrued Interest (3 days)	$543.03 (3/183 x .033125 x $1,000,000)	
Full Price	$996,636.78	
Settlement price	100-00	

Assume that this trade is executed the morning of April 2, the market rallies during the day, and the note is marked at 100-00 at 2:00 p.m. The seller is now short a bond worth $1,000,543.03 (clean price plus accrued interest). By reversing the bond in (buying) at the 2:00 p.m. price, then delivering it to the buyer at the (lower) sale price, the trader generates a settlement variation pay amount for April 3. Here's how:

The trader borrows the note overnight by lending $1,000,543.03 for April 3 (settlement day), and agreeing to sell back the note for the loan amount plus $152.86 in interest ($1,000,695.89) on the following day. On settlement day, the trader will receive the $996,636.78 selling price. The trader's Treasury account at day's end on April 2 will show a dollar deficit of $3,906.25, equal to $1,000,543.03 (reverse loan) minus $996,636.78 (short sale proceeds). The cash flows for both legs take place on April 3.

The market rally has a negative effect on the trader's Treasury note position, but a positive effect on his Eurodollar strip position. Assume the rally results in a gain on the long Eurodollar position of $3,500. The settlement variation collect for the Eurodollar side of the trade is made on April 3, the morning following the trade.

April 3 (Thursday)

The trader receives the 5-year note on the reverse and delivers it to the buyer. The trader receives the note's $996,636.78 purchase price, and wires $1,000,543.03 to the reverse counterparty. The $3,906.25 settlement variation pay is partially covered by the $3,500 settlement variation collect amount generated by the long Eurodollar position. The trader would pay variation margin of $406.25 on this day to cover the spread losses. This deficit must be financed overnight.

Let's assume that the trader buys back the short note position, and sells out of the Eurodollar futures this same day (Thursday, April 3). Assume that the sale price of the note remains 100-00, and Eurodollar future prices have not changed since the previous day's close.

Trade Date	April 3, 1997	
Settlement Date	April 4, 1997	
Bond Price	100-00	($1,000,000)
Accrued Interest (4 days)	$724.04 (4/183 x .033125 x $1,000,000)	
Full Price	$1,000,724.04	

April 4 (Friday) Settlement Day

Because of the previous day's purchase to cover the short, the trader pays $1,000,724.04 and receives the 5-year note. This note, purchased on April 3, is then sent to the reverse repo counterparty in return for $1,000,695.89 to satisfy the sale leg of the overnight reverse transaction. The Treasury position is now flat. Assuming an overnight financing rate of 5.5% (annualized), the deficit ($406.25) grows to $406.31 on April 4. Add to this the difference between the resale price on the reverse ($1,000,695.89) less the purchase price paid to cover the short ($1,000,724.04) and the result is an additional loss of $28.15. The net deficit at the end of April 4 is $434.46.

One Good Turn

By Galen Burghardt and Susan Kirshner

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Galen Burghardt and Susan Kirshner explain how taking advantage of anomalies in borrowing costs around the year end can make for a happy new year.

It has been eight years since there was last any serious pressure on year-end dollar financing rates. But 1986 lives on in people's memory because the spike in the fed funds rate that year — and the year before— was so large and so expensive for those who had to borrow. As a result, "the turn", a two-, three- or four-day period from the last business day of one year to the first business day of the next, still has a profound effect on the way people think about year-end financing.

figures 1 and 2 show both how much and how quickly the market's perceptions of the possible premium in turn financing rates can change. They chart the spread between the December and January one-month LIBOR futures prices for year-end 1992 and 1993 respectively. The December LIBOR futures price is 100 less the value of a one-month forward rate that spans the end of the year while the January futures price is 100 less the value of a one-month forward rate that does not. This means that any increase in the turn premium will decrease the value of the spread. Using the rules of thumb developed in this article, the 60-tick drop in the spread in November 1992 was evidence of a 500-basis-point increase in the turn premium. The 20-tick drop in the spread in October 1993 suggested a 200bp increase in the expected turn premium. The subsequent rise in the spread largely reflects a fading of the market's concern over the turn premium as the year drew to a close.

The purpose of this article is to explain what the turn is and how it affects spot and forward dollar interest rates that span the end of a calendar year. We also draw out the implications for trading LIBOR and Eurodollar futures and options on futures. In doing so, we derive some useful rules of thumb for translating turn premiums into futures market spreads and show how volatility in the turn premium translates into additional volatility in the December LIBOR and Eurodollar futures prices. One of the things we find is that options traders regularly seem to pay far too much for the extra volatility.

Figure 1: Libor futures calendar spreads: Dec 1992 contract vs. Jan 1993 contract

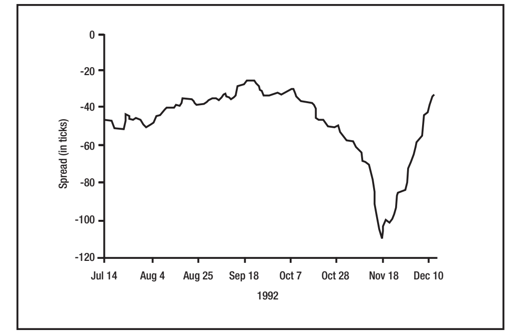
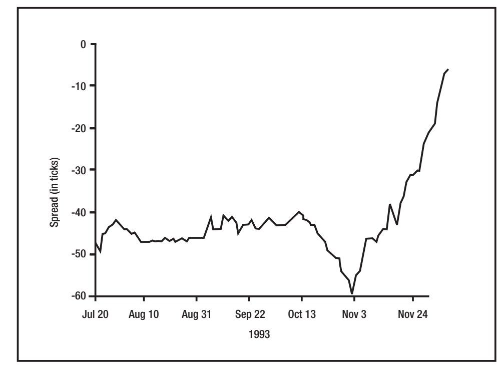


Figure 2 : Libor futures calendar spreads: Dec 1993 contract vs. Jan 1994 contract



The Turn

"The turn" is the period between the last business day of the current calendar year and the first business day of the new year. As New Year's Day is a holiday, the number of days in the turn is at least two calendar days and can be three or four.

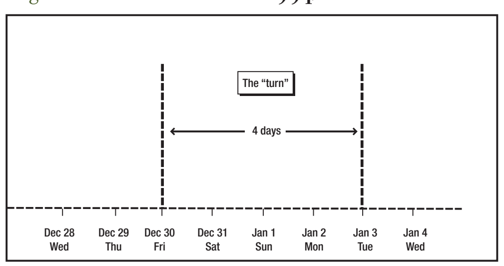
The turn lasts two days if December 31 falls on a Monday, Tuesday or Wednesday. In each of these cases, the next calendar day is a holiday so that money borrowed on Monday would be paid back on Wednesday, two days later. Money borrowed on Tuesday is paid back on Thursday, and money borrowed on Wednesday is paid back on Friday.

If December 31 falls on a Friday or Saturday, so that January 1 falls on a weekend, the number of days in the turn depends on whether the Fed wire is open on the following Monday. In 1993, December 31 fell on a Friday and the Fed wire was open on the following Monday (January 3). Since money borrowed on the Friday was paid back three days later on the Monday, 1993/1994 was a three-day turn. This year, December 31 falls on a Saturday and the Fed wire will be closed on the Monday (January 2), making 1994/1995 a four-day turn.

If December 31 falls on a Thursday, the turn will last four days since money borrowed on Thursday will be paid back the following Monday.

Figure 3 shows a time line of the turn for the end of 1994. The last business day is Friday, December 30. A bank looking to borrow overnight funds on Friday would normally repay those funds the following Monday, which is the next business day. But this year, as New Year's Day falls on Sunday, the Fed wire is closed on Monday, January 2.

Figure 3. Time line for the 1994 turn



The turn has gained notoriety among bankers because of the pressures that have been brought to bear on year-end financing rates in years past. The source of this pressure is said to be the demand by banks for cash that can be used to dress up their balance sheets at the end of the calendar year. Although the Fed does what it can to accommodate this year-end increase in demand for liquid balances, and does an excellent job most of the time, it appears to have misjudged the size of the shift at least twice since 1984.

Figure 4 shows that the turn rate and the average rate around the turn appear to have been fairly close to one another in most of the past 10 years. In 1984, for example, normal financing rates during the five days before and after the turn were around 8.37%. For the turn between 1984 and 1985, the turn rate increased to 8.74%, for a turn premium of 0.37%. The "turn ratio", which is simply the ratio of the turn rate to the non-turn rate and which we will use later when we examine the effect of the turn on rate volatility, was only 1.04.

Figure 4. Fed funds behaviour around year end

Year	Average rate around the turn (1)	Turn rate (2)	Turn premium [1] (3)	Turn ratio [2] (4)
1984	8.37	8.74	0.37	1.04
1985	8.14	13.46	5.32	1.65
1986	7.57	14.35	6.78	1.90
1987	6.96	6.89	- 0.07	0.99
1988	9.14	9.04	- 0.10	0.99
1989	8.52	7.97	- 0.55	0.94
1990	7.73	5.53	- 2.20	0.72
1991	4.25	4.09	- 0.16	0.96
1992	3.16	2.66	- 0.50	0.84
1993	3.04	2.85	- 0.19	0.94
mean	**6.69**	**7.56**	**0.87**	**1.10**
std. dev.	**2.31**	**4.05**	**2.84**	**0.37**

[1] = (2) - (1); [2] = (2) / (1)

At the end of 1985, however, the turn premium was more than five percentage points, and at the end of 1986, nearly seven. The effect of a seven percentage point turn premium on the cost of funding $1 billion over the year end, even for a turn period as short as two days, is $389,000. This is serious money in anybody's book.

Since 1986, realised rate behaviour around the turn has been unremarkable. Even so, the possibility of a large premium still looms large, and wide swings in the market's expectations about turn financing rates can have dramatic effects on forward deposit rates.

Effects on Eurodollar and LIBOR Futures Prices

As the one-month LIBOR and three-month Eurodollar futures contracts that expire in December settle to deposit rates that span the end of the year, changes in the turn rate affect the final settlement value of these two contracts. This year, for example, the December LIBOR and Eurodollar contracts expire on December 19. The final settlement price of the one-month LIBOR contract on that day will be 100 - R_{1m}, where R_{1m} is the one-month deposit rate on December 19 for the 33-day deposit period that runs from Wednesday, December 21, through Monday, January 23 (see figure 5 below). The final settlement price of the three-month Eurodollar contract will be 100 - R_{3m}, where R_{3m} is the three-month deposit rate on December 19 for the 90-day period that runs from December 21 through Tuesday, March 21.

The relationship between the turn rate and the deposit rates to which the LIBOR and Eurodollar futures contracts will settle can be determined by comparing two borrowing transactions. In the first, money is borrowed for the full term at a term lending rate. In the second, money is borrowed in three legs — one that runs from December 21 through December 29, one that runs from December 30 through January 3, and one that runs from January 4 through the end of the term. Under the first strategy for borrowing one-month money, one dollar borrowed on December 21 would call for

$$\$1\ [1 + R_{1m}\ (33 / 360)]$$

to be repaid on January 23. Under the second strategy, one dollar borrowed on December 21 would require a repayment of

$$\$1\ [1+R_b\ (D_b / 360)] \times [1+R_t\ (D_t / 360)] \times [1+R_a\ (D_a / 360)]$$

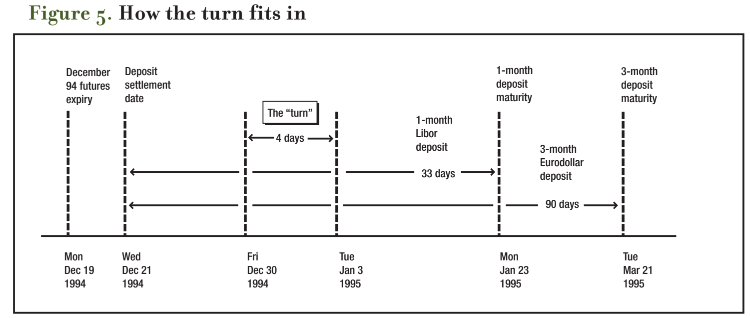
where R_b, R_t and R_a are the rates that apply to the days before, during and after the turn and where D_b, D_t and D_a are the actual number of days in the periods before, during and after the turn. For a bank financing a position over this period to be indifferent between the two strategies, the two amounts of money must be the same. If we collapse the rates before and after the turn into a single, non-turn deposit rate, R_{nt}, the two strategies cost the same if

$$R_{1m} = [(1+ R_{nt}\ (D_{nt} / 360)\ )\ (1+ R_t\ (D_t / 360)\ ) - 1]\ (360 / 33)$$

The three-month term deposit rate can be expressed the same way. The only difference is that the non-turn rate for the 90-day period would be different from the non-turn rate for the 33-day period.

To get a sense of how large an effect the turn can have on December LIBOR and Eurodollar futures prices, suppose first that the turn and non-turn rates are the same, say 6%. In this case, both one-month and three-month deposit rates would be (except for a trivial amount of compounding) 6%. December LIBOR and Eurodollar futures prices would both be 94.00 [= 100.00 - 6.00].

Figure 5. How the turn fits in



Suppose now that the turn rate increases by 200bp to 8%, while the non-turn rate stays at 6%. At these rates, and given the day counts shown in figure 5, the one-month forward deposit rate for December 19 would be

$$F_{1m} = [(1+0.060 (29/360)) (1+ 0.080 (4/360)) -1] [360/33]$$

$$= 0.0625$$

which is 25bp higher than the one-month forward rate with the turn rate at 6%. The three-month or 90-day forward deposit rate would be

$$F_{3m} = [(1+0.060 (86/360)) (1+ 0.080 (4/360)) -1] [360/90]$$

$$= 0.0609$$

which is 9bp higher than the three-month forward rate with the turn at 6%. At these rates, the fair value of the December LIBOR contract would be 93.75 [= 100.00 - 6.25], and the fair value of the December Eurodollar contract would be 93.91 [= 100.00 - 6.09]. Thus, the effect of a 200bp spread between the turn and non-turn rates is to decrease the fair value of the December LIBOR contract by 25bp and the fair value of the December Eurodollar contract by 9bp.

Although the effect of any given turn/non-turn rate spread on the fair value of the December LIBOR and Eurodollar futures contracts depends to some extent on the actual number of days in the forward periods and on the level of rates, we have what we need for excellent working rules of thumb:

- With a four-day turn, the effect of each 100bp increase in the spread between the turn and non-turn forward deposit rates is a 12-tick decrease in the fair value of the December LIBOR contract and slightly more than a 4-tick decrease in the fair value of the December Eurodollar contract.

- With a three-day turn, the effect of each 100bp increase in the spread between the turn and non-turn forward deposit rates is a 9-tick decrease in the fair value of the December LIBOR contract and just over a 3-tick decrease in the fair value of the December Eurodollar contract.

- With a two-day turn, each 100bp increase in the spread between the turn and non-turn forward deposit rates reduces the fair value of the December LIBOR contract by about 6 ticks and the fair value of the December Eurodollar contract by just over 2 ticks.

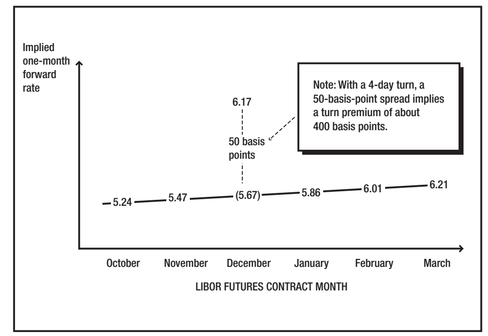
Figure 6. Effect of turn rates on the fair values of December 1994 LIBOR and Eurodollar futures prices (four-day turn)

	LIBOR futures			Eurodollar futures		
	Non-turn rates			Non-turn rates		
Turn rate	3%	5%	7%	3%	5%	7%
3%	0	24	49	0	9	18
5%	-24	0	24	-9	0	9
7%	-49	-24	0	-18	-9	0
9%	-73	-49	-24	-27	-18	-9
11%	-97	-73	-49	-36	-27	-18
13%	-122	-97	-73	-45	-36	-27

These rules of thumb are borne out by figure 6, which shows the effect of various rate spreads on the fair value of both the December LIBOR and Eurodollar futures contracts given a four-day turn. For example, if the non-turn forward deposit rate were 5% and the turn rate 11%, the effect of the 600bp spread would be a 73-tick reduction in the fair value of the December 1994 LIBOR futures contract. The same spread would produce a 27-tick reduction in the fair value of the December 1994 Eurodollar futures contract. As the effect of the turn rate is roughly proportional to the length of the turn, the effects of these rate spreads given two-day and three-day turns can be determined easily enough from the data in figure 6.

With these rules of thumb, it is easy to get a reading on the spread between turn and non-turn rates by looking at the pattern of rates implied by the one-month LIBOR contracts, which have serial expirations extending out 12 months at any one time. On October 3, 1994, for example, there were one-month LIBOR futures with expirations ranging from October 1994 through September 1995. Figure 7 shows the strip of one-month forward deposit rates implied by their October 3 settlement prices. The effect of the turn on the pattern of rates stands out clearly. The one-month deposit rate for the November contract, which spans the period from mid-November to mid-December, was 5.47%. The one-month deposit rate for the January contract, which spans the period from mid-January to mid-February, was 5.86%. In between, the one-month deposit rate for the December contract was 6.17%, about 50bp higher than the 5.67% that the surrounding rates would suggest for a one-month December deposit rate.

Figure 7. Implied one-month forward deposit rates on October 3, 1994



From this 50bp differential, we can determine the spread between turn and non-turn financing rates that is implied by the LIBOR futures contract. Using the rule of thumb that each 100bp in the spread reduces the fair value of the December LIBOR contract by about 12bp, the 50bp differential in the December contract implies a spread of about 400bp between the turn and non-turn rates.

This implied rate spread can be compared easily with the spreads quoted in the forward deposit market as a way of comparing the pricing of the turn in the two markets. If you find, for example, that the implied turn rate differential is larger than the actual, then you know that the December LIBOR contract is cheap relative to cash.

Implications for Futures Spreads

As the turn rate affects both the December LIBOR and Eurodollar futures contracts, it affects the values of several key futures spreads including the:

- **December LED spread** In this spread, you are long the LIBOR contract and short the Eurodollar contract. Given the rule of thumb for a four-day turn, each 100bp increase in the turn premium translates roughly into an 8-tick decrease in the value of this spread. Thus, the December LED spread is about 32 ticks lower than it would be if the turn premium were zero.

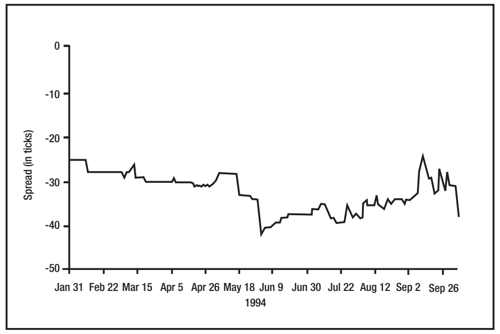
- **December/January LIBOR spread** In this spread, you are long the December and short the January LIBOR contracts. As the turn premium affects only the December contract, each 100bp increase in the turn premium is worth about 12 ticks in this spread. As shown in figure 7, this spread is 50 or so ticks lower than it would be if there were no turn premium.

- **December/March Eurodollar spread** Here you are long the December and short the March Eurodollar contracts. With a four-day turn, each 100bp increase in the turn premium decreases the value of this spread by about 4 ticks.

- **December TED spread** In this spread, you are long the three-month December Treasury bill contract and short the December Eurodollar contract. As you are short the Eurodollar contract, each 100bp increase in the turn premium increases the value of this spread by about 4 ticks. One can work out similar implications for the November/ December/January LIBOR butterfly and the December/March TED tandem.

Of the various spreads, the December/ January LIBOR spread is one of the better vehicles for trading the turn because the effect of the turn premium on the December contract price is both large and fairly direct, and the calendar risk in the trade is about as small as it can be without actually trading the cash deposits themselves. If the turn premium falls to zero by the time the December LIBOR contract expires on the 19th, a long position in the spread would gain 50 ticks, or $1,250 per spread. Figure 2 shows how this spread behaved last year, and Figure 8 shows how the spread has performed so far this year.

Figure 8. LIBOR futures calendar spreads: Dec 1994 contract v Jan 1995 contract



The dangers in this spread are threefold. One is that the turn rate is not realised until two weeks after the December contract expires. A second is that there is considerable fluctuation in the market's perception of the turn premium throughout the months leading up to the end of the year. A third is that you are exposed to a flattening of the near-term yield curve.

The other spreads may be less attractive for trading the turn premium, but the effect of the turn on them cannot be ignored when evaluating trades that involve them. The December TED spread, for example, as well as the December/March TED tandem, are greatly influenced by the turn premium. With an implied turn premium of around 400bp, December Eurodollar futures trade 16 ticks or so lower than they would without the turn. Thus, we know that about 16 ticks of the current December TED spread can be attributed to the turn. By the same token, the December/March Eurodollar calendar spread is 16 ticks lower than it would be without the turn.

Effect of the Turn on LIBOR and Eurodollar Volatilities

Uncertainty about financing rates over the turn is an additional source of volatility for the one-month and three-month deposit rates to which the December LIBOR and Eurodollar futures contracts will settle. How can the effect of turn-rate volatility on the volatilities for options on December LIBOR and Eurodollar futures be determined?

The biggest hurdle to calculating this is how to represent turn-rate volatility. The few observations that we have on the turn, which are shown in figure 4, suggest fairly strongly that turn rates are not lognormally distributed.

With so few observations, however, we only have what we think are two reasonable guides to choosing an alternative distribution. The first is that the size of the turn rate premium should be related to the level of interest rates. The second is that the chance of getting a huge turn premium should be fairly large even though most turn premiums will be close to zero.

One way to satisfy the first reasonableness check is to allow the ratio of the turn rate to the non-turn rate to be the random variable so that the turn premium is directly proportional to the level of rates. For example, a turn/non-turn ratio of 1.5 would produce a turn rate of 9% if base rates were 6%. If the base rate were 3%, the turn rate would be 4.5%. The turn premium in the first case would be 3%, while the turn premium in the second case would be 1.5%.

We can satisfy the second reasonableness check by allowing the behaviour of the ratio of turn to non-turn rates to be described by the gamma distribution, which has fat enough tails to allow for a comparatively high number of very large outcomes.

Using this approach, we can simulate the distribution of the one-month and three-month deposit rates that span the turn using various levels of volatility for non-turn rates and for the turn/non-turn ratio. From these simulated distributions, we can determine the effect that turn-rate volatility should have on the volatility of the December one-month LIBOR and three-month Eurodollar futures contracts. The results of these simulations are shown in figures 9 and 10.

Figure 9. Add-on turn volatility premium (3% forward rate)

Volatility of the turn ratio	Base rate volatility for: One-month LIBOR Two-day turn		Four-day turn		Three-month Eurodollars Two-day turn		Four-day turn	
	15%	25%	15%	25%	15%	25%	15%	25%
25%	0.40	0.56	0.86	1.10	0.33	0.53	0.68	1.04
35%	0.48	0.60	1.12	1.25	0.35	0.54	0.76	1.08
45%	0.62	0.68	1.51	1.49	0.39	0.56	0.90	1.17

Figure 10. Add-on turn volatility premium (6% forward rate)

Volatility of the turn ratio	Base rate volatility for: One-month LIBOR Two-day turn		Four-day turn		Three-month Eurodollars Two-day turn		Four-day turn	
	15%	25%	15%	25%	15%	25%	15%	25%
25%	0.72	1.05	1.46	2.00	0.65	1.04	1.30	1.99
35%	0.83	1.11	1.81	2.20	0.69	1.05	1.51	2.07
45%	1.02	1.23	2.33	2.52	0.76	1.10	1.69	2.21

How much is turn-rate volatility worth for options on December LIBOR and Eurodollar futures? The results shown in figures 9 and 10 suggest that it should be fairly small. Consider the case in which the volatility (ie, standard deviation) of the turn/non-turn ratio is 0.45; forward deposit rates are around 3%; the base rate volatilities of one-month LIBOR and three-month Eurodollar rates are 25%; and the turn period lasts four days.

As shown in figure 9, the contribution of volatility in the turn rate would add 1.49% to the volatility of the December LIBOR futures contract, and 1.17% to the volatility of the December Eurodollar futures contract. The contribution is smaller for lower levels of volatility. And, at any given set of volatilities, the contribution is smaller for a two-day turn than for a four-day turn.

The effect of turn-rate volatility is higher if the level of non-turn interest rates is higher. This is shown in figure 10, where everything is the same as in figure 9 except that the level of non-turn rates is 6% rather than 3%. At this level of rates, we reckon that the effect of 0.45 volatility in the turn/non-turn ratio combined with 25% base rate volatility is an increase of 2.52% in December LIBOR and 2.21% in December Eurodollar volatility for a four-day turn.

These results may not seem very exciting at first glance because they cannot shed much light on whether December LIBOR or Eurodollar options are expensive or cheap. But they can be a powerful tool in evaluating spread trades between December LIBOR and Eurodollar options.

For instance, even in the extreme case — rates at 6%, turn ratio volatility at 0.45, base rate volatility at 25% and a four-day turn — the effect of turn-rate volatility on the difference between LIBOR and Eurodollar volatilities would only be about 0.3% (the difference between 2.52 and 2.21, as shown in figure 10). In less extreme cases, and with a three-day turn, the effect would be smaller.

Figure 11. LED volatility spreads: December 1992 contracts

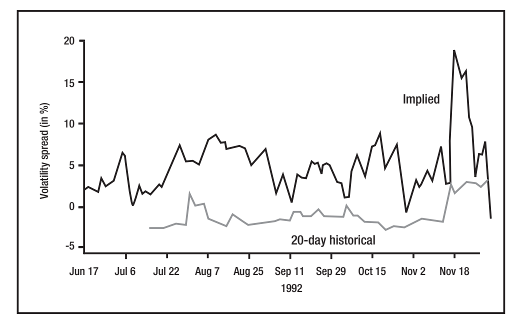


Figure 12. LED volatility spreads: December 1993 contracts

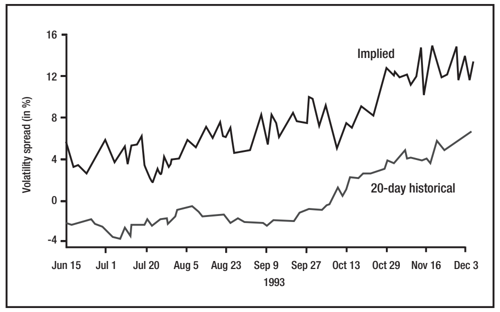
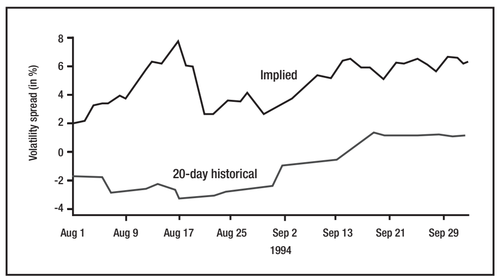


Figure 13. LED volatility spreads: December 1994 contracts



On this basis, we would expect the spread in implied volatilities for the LIBOR and Eurodollar options to be quite small. But in figures 11, 12 and 13 we see that in 1992, 1993 and 1994 the options market has paid a hefty premium for the LED volatility spread. In 1993, for example, the implied volatility spread was consistently about 8% greater than the historical volatility spread. At the time of writing, the LED-implied volatility spread for the December 1994 contracts is trading around 6.5% — about 5% greater than the historical volatility spread.

We view this as an opportunity to take advantage of an apparent mispricing. For example, to sell December 1994 LIBOR volatility and buy December 1994 Eurodollar volatility on October 3, 1994, one could have:

- sold 100 of the December 94.00-93.75 LIBOR strangles at 35 ticks per strangle and sold 10 December LIBOR futures to make the position delta neutral (futures at 93.83) and
- bought 100 of the December 94.00 Eurodollar straddles at 30 ticks per straddle and bought 11 December Eurodollar futures to make the position delta neutral (futures at 93.96).

Thus, the spread position could have been established for a net credit of 500 ticks.

A position like this would have some interesting and desirable characteristics. As the spread is long the low-volatility options and short the high-volatility options, the net position provides a rare opportunity to be long gamma and to have time decay work in your favour at the same time.

Figures 11 and 12 show how highly variable the implied LED volatility spread is. Thus, even though the additional premium paid for LIBOR volatility seems not to be justified by either the theory or the evidence, a position that is short LIBOR volatility and long Eurodollar volatility can produce large swings in a trader's profit and loss from day to day. Also, a sharp increase in the turn rate can be costly for anyone who is short LIBOR volatility. In late November 1990, for example, such a spike in the turn rate increased the 20-day historical volatility spread to around 14%.

Even so, there are two ways the trader can make money on the trade. The first is a collapse in the implied volatility spread so that it accords more closely with what it should be. This is the best outcome because it avoids the need to actually work for a living by managing the position until the December expiration of the options.

If the implied volatility spread does not collapse, the trader can still make money if the realised difference between December LIBOR and Eurodollar volatilities proves to be less than 6.5%. In this case, if the position is properly managed, the trader can profit from the relatively higher time decay that would be taken in on the LIBOR options than would be paid out on the Eurodollar options.

Galen Burghardt is a senior vice-president and Director of Research at Carr Futures in Chicago.

Eurodollar Bundles and Packs

by Frederick Sturm

Right from their introduction in September 1994, Eurodollar bundles have proven to be a powerful and convenient tool for those who deal in strips of ED futures contracts. Since then, the original bundle concept has been expanded to include 1-, 2-, 3-, 4-, 5-, 6-, 7-, 8-, 9-, 10-year bundles and 5-year "forward" bundles. In this paper we spell out what ED bundles are and how they work.

What Are ED Bundles?

An ED bundle is the simultaneous sale or purchase of one each of a series of consecutive ED contracts. The first contract in any bundle is generally the first quarterly contract in the ED strip, though since October 1998, bundles can be constructed starting with any quarterly contract. The leading exception to this convention is the 5-year "forward" bundle, which covers years 5 through 10 of the Eurodollar futures strip. For example, on April 11, 2000, the first contract in the 5-year "forward" bundle would be June 2005 (the 21st quarterly contract in the strip), and March 2010 (the 40th contract) the last.

The terminal contract depends upon the bundle's term to maturity. The CME offers 1-, 2-, 3-, 4-, 5-, 6-, 7-, 8-, 9- and 10-year terms to maturity. The specifics for each term as of April 11, 2000 appear in the first three columns of the table below.

Bundle Features

Term to Maturity	Comprising One Each of	Terminal Contract	DV-01 ($)	DV-Tick ($)
1-year	first 4 contracts	March 2001	100	25
2-year	first 8 contracts	March 2002	200	50
3-year	first 12 contracts	March 2003	300	75
4-year	first 16 contracts	March 2004	400	100
5-year	first 20 contracts	March 2005	500	125
6-year*	first 24 contracts	March 2006	600	150
7-year	first 28 contracts	March 2007	700	175
8-year*	first 32 contracts	March 2008	800	200
9-year*	first 36 contracts	March 2009	900	225
10-year	first 40 contracts	March 2010	1,000	250
5-year "fwd"	last 20 contracts	March 2010	500	125

* 6-, 8- and 9-year Bundles were first listed on April 17, 2000.

Making Prices in Bundles

For any bundle, the price will be quoted in terms of net change during the current trading session from the previous trading day's settlement level. Specifically, the bundle's price quotation will reflect the simple average of the net price changes of each of the bundle's constituent contracts.

Example 1: A trade is executed in the 2-year bundle at a price quotation of –1. This reflects an agreement between the buyer and seller that among the nearest eight ED contracts (for example the June '00 ED to the March '02 ED) the average net change in the contracts' prices (versus their price levels at yesterday's settlement) is minus one tick.

Example 2: Assume that all of the nearest 21 contracts (e.g., the June '00 ED to the June '05 ED) have enjoyed a three-tick increase in the price since yesterday's settlement; at the same time the prices of each of the next seven contracts (e.g., the September '05 ED to the March '07 ED) have posted net gains of four ticks. Under these conditions, the implied fair-value price quotation for the 7-year bundle would be:

$$\frac{[(21 \times 3) + (7 \times 4)]}{28} = 3.25 \text{ ticks}$$

Example 2 raises a critical point. Bundle prices are quoted in increments of one quarter (1/4) of a basis point.

For ED bundles the value of .01 will always be four times greater than the value of the quarter tick minimum price increment. These differences are summarized in the previous table.

Un-bundling After the Trade

After a buyer and a seller have agreed upon the price and quantity of a bundle, they must assign mutually agreeable prices to each of the bundle's constituents. In principle, the traders may set these component prices arbitrarily, subject to one restriction: the price of at least one constituent ED contract must lie within that contract's trading range for the day (assuming that at least one of the ED contracts in the bundle has established a trading range). CME regulations are designed this way to ensure that bundle prices will remain tethered to the price action of the underlying individual ED contracts.

In the vast majority of cases, traders and clearing firms make use of a computerized system that automatically assigns individual prices to the contracts in a bundle. This system was designed by CME to simplify the administrative aspects of the bundle trade.

The pricing algorithm used by CME is based upon the following principle: To the extent that adjustments are necessary to bring the average price of the bundle's components into conformity with the bundle's traded price, these price adjustments should begin with the most deferred ED contract in the bundle and should work forward to the nearest ED contract. The following example illustrates the application of these principles.

Suppose that a buyer and a seller who are transacting in the 3-year bundle have agreed upon a net price change of –2.5 basis points (bps) versus the previous day's settlement level. Suppose, moreover, that the day's actual net price changes for the bundle's constituent ED contracts are as follows: –2 bps for the nearest eight contracts and –3 bps for the next four contracts. The implied average price change is [(8 x -2) + (4 x -3)] /12 = –2.33 bps, which exceeds the bundle's price change by one sixth of a basis point.

The CME algorithm would resolve this disagreement in two steps, dealing first with the integer portion of the –2.5-tick trade price (the 2), and then with the fractional portion (the 0.5). Specifically, the algorithm would begin by assigning to each of the twelve contracts in the bundle a net price change of –2 ticks from the previous day's close. Then it would adjust these price changes downward, proceeding one contract at a time, beginning with the bundle's terminal contract and working forward — until the average net price change for the bundle is the agreed-upon –2.5 ticks. Following this procedure would result in net price changes of –2 ticks for the bundle's six nearest contracts and –3 ticks for its six most deferred contracts. The result is an average price change of (6 x -2 + 6 x -3) / 12 = –2.5 bps, as desired.

Simple to Structure, Simple to Execute

By construction, bundles are well-suited to traders and investors who deal in LIBOR-based floating-rate products. Obvious examples include investment banks that routinely carry syndication inventories of floating-rate notes, corporate treasuries that issue floating-rate debt, or commercial bankers who wish to hedge the risk exposure entailed in periodic loan-rollover agreements.

However, bundles' most avid followers are likely to be those market participants who deal in long-dated Treasury-Eurodollar (TED) spreads. Such trades entail the purchase (or sale) of a Treasury security and the simultaneous sale (or purchase) of a strip of Eurodollar futures contracts with a comparable notional term to maturity. A frequently encountered version comprises a long position in the two-year Treasury note and a short position in some combination of the nearest seven or eight ED contracts.

Despite their popularity, until the introduction of Eurodollar bundles such transactions suffered for lack of any generic standard. Bond dealerships that promote long-dated TED spreads to their clients tend to recommend trades that involve odd numbers of Eurodollar contracts, differing from one point in the ED strip to the next. The dealers customarily justify their formulations by appealing to proprietary yield-curve models. These models purport to link the future spot interest rates that are represented by the Eurodollar futures strip to the implied zero-coupon yield curve that is embedded in the prices of U.S. Treasury securities.

Most such yield-curve models produce speciously precise results: too often, mathematical interpolation of painstaking exactitude sits cheek-by-jowl with broad, crude assumptions about the actual shape of the term structure of the Treasury-to-Eurodollar yield spread. Unfortunately, this is precisely the actuarial risk — a default by an off-U.S.-shore commercial bank on its liabilities (versus the default-free character of the U.S. Treasury's debt) — that is at the very heart of the TED-spread trade and that imparts any value to it.

In this context, CME's introduction of evenly weighted bundles of Eurodollar contracts serves two useful ends. First, it establishes a readily available, widely acceptable, and easily interpretable benchmark by which the performance of any other TED trade can be judged.

Second, and more importantly, it facilitates cleaner, more rapid execution of the Eurodollar side of the trade. Instead of being forced to construct lengthy and idiosyncratic ED strip positions contract by contract (always a risky proposition, especially in a fast-moving market), the futures broker now has the capability of executing one trade on behalf of the client that establishes price, quantity, and the allocation of the bundle's component prices.

Eurodollar Packs

1. What are Eurodollar packs?
Packs are the simultaneous purchase or sale of an equally weighted, consecutive series of four Eurodollar futures, quoted on an average net change basis from the previous day's close. This quoting method is similar to that for Eurodollar bundles.

2. Why packs?
Packs are an alternative method of executing a strip trade. All four contract months in the strip are executed in a single transaction, eliminating the inconvenience of partial fills, particularly in the deferred contract months.

3. How many are there?
Packs, like Eurodollar futures, are designated by a color code that corresponds to their position on the yield curve. The most commonly traded Packs[1] are the Red, Green, Blue, Gold, Purple, Orange, Pink, Silver, and Copper[2], corresponding to Eurodollar futures years 2-10, respectively. "Pack spreads" (e.g., Pack butterflies and individual contract months versus Packs) are also listed for trading.

4. How are packs quoted?
Packs are quoted in minimum one-quarter-basis point (.25) increments, e.g., + 2.5/bid + 2.75/ask.

5. How are prices assigned to individual legs?
Any price consistent with the agreed-upon average net change for the pack at which the trade occurred may be assigned to the individual legs. Because of this, packs are classified as spread trades. Any prices can be assigned to individual contracts, as long as they are consistent with the agreed-upon average net change. Whole-basis-point prices are assigned to individual legs of the Pack consistent with the traded price. For example, if a pack is down is down 2.25 ticks, then individual contracts will be –2, –2, –2, and –3.

This article was originally written by Frederick Sturm, First Vice President, Fuji Securities in 1994. Frederick Sturm is now Senior Economist at the Chicago Board of Trade. The article was recently updated by Peter Barker, Director, Interest Rate Products at Chicago Mercantile Exchange.

1 "Rolling" Bundles and Packs were introduced in October 1998. These Packs and Bundles can be constructed starting with any quarterly Eurodollar expiration (provided of course, that the last contract in the combination does not extend beyond the last contract.)

2 There is a one-day delay in listing a new 10-year Bundle and Copper Pack after the expiration of the front quarterly futures contract. This is due to the one day lag between expiration of the front quarterly contract and the listing of the 40th quarterly Eurodollar contract.

Eurodollar Mid-Curve Options: A User's Guide

by Peter Barker

What Are Mid-Curve Options?
Mid-Curve options are short-dated American-style options on long-dated Eurodollar futures. These options, with a time to expiration of three months to one year, have as their underlying instrument Eurodollar futures one or two years out on the yield curve. Hedging and trading opportunities are created on the mid-range of the yield curve, hence the name "Mid-Curve" options. Because the options are short-dated, they offer a low premium, high time decay option alternative for trading this part of the curve.

Mid-Curve Options Listing and Expiration
CME lists both one-year and two-year Mid-Curve options. For one-year Mid-Curves, two serial months, and four quarterly expirations are listed at a given time. The nearest two months of the March quarterly cycle only are listed for two-year Mid-Curves. These options are not cash settled, rather they exercise into a Eurodollar futures contract expiring in one or two years. The addition of Mid-Curve options means that the two serial months, and first four quarterly calendar months will each contain several option expirations: The traditional quarterly option expiration which coincides with the termination of futures trading, and a separate and distinct expiration for Mid-Curve options.

For example, March 2001 will contain two different Eurodollar option expirations. On Friday, March 16, 2001, the March one-and two-year Mid-Curve options, with respective underlying contracts the March 02 and March 03 Eurodollar futures, will expire. On Monday, March 19, 2001, Eurodollar quarterly options will expire with the March 01 Eurodollar as the underlying contract.

On the first Exchange business day following a nearby Mid-Curve options expiration, the next Mid-Curve option in the cycle will be listed. For example, upon expiration of the May 01 serial Mid-Curve on Friday, May 11, 2001, the August 01 serial will be listed on the following Monday, May 14. It will then join the already existing July 01 serial, and the June 01, September 01, December 01, and March 02 one-year Mid-Curves, and the June 01 and September 01 two-year Mid-Curves.

Termination of Trading for Mid-Curve Options
Like traditional serial month Eurodollar options, Mid-Curve options deliver futures contracts upon exercise and trading terminates at the close of trading on the Friday preceding the third Wednesday of the contract month. If that date is an Exchange holiday, Mid-Curve options trading will terminate on the immediately preceding business day. This is the same Friday expiration cycle for serial month Eurodollar options.

First: A Review of Some Mid-Curve Options Basics
The following section illustrates important concepts that all hedgers and traders must understand before they use options of any kind. First, you'll find definitions and a discussion of the attributes of these options derivatives or risk parameters commonly referred to as "the Greeks." These option derivatives will be defined and discussed including: delta, gamma, theta and vega. Then, to better understand these concepts, there are illustrations of "trading the gamma" and of constructing a trade based on the outlook for the yield curve.

Options Risk Parameters- "The Greeks"

Delta
Definition: the change in the price of an option for a one-point change in the price of the underlying futures.

- Probability that the option will end up in-the-money
- Hedge ratio: the number of futures contracts that one option can hedge
- The change in delta as an option goes in/out-of-the-money becomes more pronounced as the option approaches expiration
- The deltas of both in-the-money and out-of-the-money options tend toward .50 as volatility increases
- Delta is related to:
  - time to expiration
  - level of volatility
  - the amount by which the option is in/out-of-the-money

Gamma
Definition: the change in an option's delta for a one-point change in the price of the underlying futures.

- An at-the-money option has the greatest gamma
- A change in volatility has the greatest effect on the gamma of an at-the money option
- As expiration nears…
  – gamma of at-the-money options approaches infinity
  – gamma of in/out-of-the money options decreases toward zero

Vega
Definition: the change in the price of an option given a one percent change in the volatility of the underlying futures

- Greatest for at-the-money options
- Decreases as an option moves in/out-of-the-money
- Increases as time to expiration increases
- Is always positive

Theta
Definition: the change in the value of an option for a one day decrease in the time remaining to expiration.

- An at-the-money option has the greatest theta
- Decreases as the option moves in/out-of-the-money
- For at-the-money options, theta increases as time to expiration decreases
- Increases as volatility increases

Example:
June '97 1-year Mid-Curve vs. June '98 Quarterly Options

Delta:	the change in delta for Mid-Curves as they go in/out-of-the-money is more abrupt
Gamma:	the gamma of at-the-money Mid-Curves is greater
Vega:	the Mid-Curves sensitivity to changes in volatility is lower
Theta:	for at-the-money Mid-Curves, theta is greater

Strategy 1

- A standard options trade involves purchasing an at-the-money straddle and subsequently "trading the gamma" of the position.
- Mid-Curve options offer a new way to trade gamma on the Red and Green contracts.

Example: May 9, 1997

	June 1997 93.50 1-year Mid-Curve Straddle	June 1998 93.50 Straddle
Price:	.27 ($675)	.75 ($1,875)
Delta:	0	-.06
Gamma:	2.32	.747
Theta (7-day):	.028 ($70.50)	.0058 ($14.50)
Vega:	.016 ($40.00)	.0512 ($128.00)
Implied Vol:	16.88%	14.65%
Expiration:	6/13/97	6/15/98
Days to Expiration:	35	402
June 1998 ED price	93.49	

Opportunity
Create a steepening yield curve position using Mid-Curve options. Take advantage of relatively wide volatility spreads and minimize premium cost.

Strategy 2

- Buy the Sep '97 (quarterly) 94.00 call
- Sell the Sep '97, 1-year Mid-Curve call
- Premium paid: 0
- Volatility spread: 6.18%

Example: May 12, 1997

	Sep97 94.00 calls	Sep97, 1-year Mid-Curve 93.75 calls
Action:	Buy	Sell
Price:	.11 ($275)	.11 ($275)
Delta:	.41	.28
Gamma:	1.09	.539
Theta (7 days):	.0037 ($9.25)	.0057 ($14.25)
Vega:	.016 ($34.25)	.0127 ($31.75)
Implied Vol:	9.91%	16.09%
Expiration:	9/15/97	9/12/97
Days to Expiration:	126	123
Sep 1997 futures:	93.94	
Sep 1998 futures:	93.43	
3-month LIBOR:	5.8125	
FOMC Meeting:	May 20, 1997	

Example: May 22, 1997

	Sep97 94.00 calls	Sep97, 1-year Mid-Curve 93.75 calls
Position:	Long	Short
Price:	.09 ($225)	.075 ($187.50)
Delta:	.40	.23
Gamma:	1.25	.559
Theta (7 days):	.0035 ($8.75)	.0064 ($16)
Vega:	.0131 ($32.75)	.014 ($35)
Implied Vol:	8.95%	14.60%
Expiration:	9/15/97	9/12/97
Days to Expiration:	116	113
Sep97 futures:	93.945	3-month LIBOR: 5.8125
Sep98 futures:	93.39	

Final Results

Market Direction:	Steady
Yield Curve:	Steepened
Net Delta:	Longer (.17 from .13)
Net Gamma:	Longer (.691 from .551)
Theta:	More Negative ($7.25/7days from $5/7days
Vega:	Shorter ($2.25 from $2.50)
Profit	1.5 ticks

Hedging with Mid-Curve Options

The unique structure of Mid-Curve options provides exposure to long-dated interest rates at a lower price than a vanilla cap, or a traditional CME quarterly option. This is because Mid-Curve options have a relatively short life (12 months maximum); thus, the buyer pays for less time premium. Though only paying for typically three to six months of time and volatility, the user gets exposure to the Eurodollar strip out to two years. Their relatively short life also means that Mid-Curves exhibit faster time decay, and greater gamma than longer-dated caps or exchange-traded options. The result is a cost-effective method of taking a relatively short term view on interest rates one or two years out on the yield curve.

For example, a 2-year Sep 99 Mid-Curve option listed on March 15, 1999, expires on September 10, 1999. The futures contract that the option tracks is the Sep 2001 Eurodollar contract, expiring on September 17, 2001, two years later.

Similarly, a 1-year Sep 99 Mid-Curve option listed on September 14, 1998, also expires on September 10, 1999. The futures contract underlying this option is the Sep 2000 Eurodollar, expiring on September 18, 2000, one year later.

The low-cost flexibility afforded by Mid-Curve options makes them a great tool for managing LIBOR-based assets and liabilities. Mid-Curves can be combined with traditional quarterly or serial Eurodollar options, allowing users to create structures that reflect very specific opinions. The liquidity of CME interest rate options makes modifying positions simple and inexpensive.

Because Mid-Curve options are priced off 3-month forward rates with relatively distant (one- and two-year) starting dates, it is important to make a couple of points about the underlying futures contracts. First, as the yield curve flattens (steepens), deferred forward rates will generally drop (rise) more quickly than spot rates, or forward rates with earlier start dates. Second, 3-month rates one and two years forward are closely, though by no means perfectly, correlated with the one- and two-year term rates. Research suggests that correlation between a given term rate and the implied futures rate spanning the final three months of that term rate are reasonably high and stable. Recent experience has shown how volatile correlation between markets can sometimes be. However, the link between forward (futures) rates and term rates has shown itself to be sufficiently robust to make Mid-Curve options useful short term hedging tools for one- and two-year interest rate exposures.

Hedging Interest Rate Risk with Mid-Curve Options

Perhaps the best way to show how Mid-Curve options work is to use a simplified example of a bank facing a period with assets repricing before liabilities, and looking to hedge against a decline in rates one year out on the yield curve. (The example on the next page uses actual price data from Spring 1999).

Assume the case of a bank on March 1, 1999, that will have assets repricing off 12-month LIBOR on April 15, 1999. Let us also assume that the bank's balance sheet is structured in such a way that these assets are funded by longer-term liabilities. The bank's risk in this case is that interest rates will fall prior to the repricing. If interest rates fall between March 1 and April 15, the bank's interest rate margin, and therefore net interest income earned on those assets, will decrease. To maintain current levels of net interest income, the bank is interested in a 45-day hedge against a fall in one-year interest rates.

March 1: Hedge Initiated

On March 1, 1999, LIBOR rates and Eurodollar prices were the following:

LIBOR	Futures	Asset Rates
1-month 4.9650%	EDM9 94.785 (5.215%)	1-yr LIBOR 5.3800%
3-month 5.0275%	EDM0 94.210 (5.790%)	
6-month 5.1225%		

That same day, CME Mid-Curve options were trading at the following levels:

Option	Price	Delta	Vol	Expiration	Underlying Future
Jun 94.50 1-yr. Mid-Curve Call	.11	.30	19.2%	6/11/99	EDM0

Our strategy is to protect ourselves from declining one-year rates by purchasing a one-year Mid-Curve options contract. Before proceeding, the hedger must be aware that using a 1-year Mid-Curve option to hedge a 1-year exposure is a cross hedge. As was stated earlier, the value of the 1-year Mid-Curve option tracks the implied rate on a 3-month rate which is set approximately one year in the future. Even though these rates generally move closely together, term rates and forward rates are not perfect substitutes for each other. Over time, the rates may diverge, and have a material affect on the results of the hedge. Our example will illustrate that a hedger using Mid-Curves to hedge term rates must be conscious of the basis risk being incurred.

The next step in this process is to determine the number of options to buy. Generally speaking, a manager tailors the size of a hedge so that the dollar value of a one basis point change in yield in the instrument being hedged is offset by an equal change in the price of the hedging instrument. In our example, the dollar value of a basis point in our asset is equal to $101.67 ($1,000,000 x .0001 x 366/360). The dollar value of a basis point for Mid-Curve options is fixed at $25.00 ($1,000,000 x .0001 x 90/360).[1] This means that the hedger must purchase four Mid-Curve calls for each $1,000,000 in notional value of the asset. In our example, we would buy four June 94.50 1-year

1 It is worth nothing that a Mid-Curve option, if traded as an outright, may trade in 1/2 tick increments (.005 = $12.50) if the option's overall premium is 5 ticks (.05) or less. However, any Mid-Curve option premium can settle in 1/2 ticks.

Mid-Curve calls at .11 ($275) per option, or a net debit in dollar terms of $1,100 ($275.00 per contract x 4 contracts). In this way, a one basis point change in the yield implied by the Mid-Curve option will equal $100 (4 contracts x $25 per contract). Because it is not possible to trade partial contracts, this $100 BPV (Basis Point Value) is as close as a hedger can get to the $101.67 BPV for the asset.

The risk/return profile of the Mid-Curve call is similar to that of a traditional long Eurodollar call position with a June 11, 1999, expiration date. The hedge makes money as implied volatility rises, and/or there is a significant decrease in the rate implied by the June 2000 futures contract. The hedge is negatively affected by time decay and a decrease in implied volatility. At expiration, the hedge breaks even at a price of 94.61 for the June 2000 futures, which exactly covers the .11 premium paid per option. The $1,100 premium paid (plus transaction costs) is the maximum loss the hedger can incur, and this amount must also be taken into account when hedging profits or losses are calculated.

April 15: Hedge Lifted

A month-and-a-half later, on April 15, 1999, interest rates were at the following levels:

LIBOR	Futures	Asset Rates
1 month 4.92875%	EDM9 95.010 (4.990%)	1-yr LIBOR 5.21375%
3 month 5.00000%	EDM0 94.580 (5.420%)	
6 month 5.03938%		

The 12-month LIBOR has declined nearly 17 basis points from 5.3800% to 5.21375%, and the yield curve has flattened substantially. Declining rates will eventually decrease the cost of bank funding, but not before reducing the income generated by the newly repriced asset. However, the long exposure out one year on the Eurodollar futures strip created by the Mid-Curve call position mitigates the negative effects of the declining rates. The option prices on April 15, 1999, were the following:

Option	Price	Delta	Vol
Jun 94.50 1-yr Mid-Curve Call	.175	.58	15.4%

On April 15, 1999, there were still 57 days until the June Mid-Curve option expiration. On that day, the 1-year Mid-Curve option position had a profit of $162.50 per contract [(.00175-.0011) x $1,000,000 x 90/360]; for a total profit of $650 (4 contracts x $162.50).

Let us assume that as expected on April 15, $1 million of assets were priced at a 12-month LIBOR of 5.21375%, and the option hedge was lifted. The future dollar return, including hedge gains, for the assets priced on April 15, 1999, would have been:

Rate on 1-year Asset = 1-year LIBOR (5.21375%)		
.0521375 x $1,000,000 x 366/360	=	$53,006
	+ HEDGE PROFIT	$ 650
		$53,656

If no hedge had been implemented, the 1-year asset would have paid the bank $53,006 at maturity in April 2000. This compares with the $54,697 (.05380 x $1,000,000 x 366/360) that the asset would have paid had rates not fallen from their March 1, 1999 levels. The difference between the two amounts, $1,691, is only partially compensated for by the $650 hedging profit. Basis risk, the fact that an out-of-the-money call was purchased, lost time premium, and (particularly in this case) a decline in implied volatility all played a part in explaining the difference between the hedge profits and the change in the asset's future cash flow resulting from the decline in interest rates.

The effect of basis risk is shown by the fact that the price of the June 2000 futures contract increased by 37 basis points over the time period covered in the example, while 12-month LIBOR declined by only about 17 basis points. The option position did not capture the full 37 basis point move because it was 29 basis points out-of-the-money when it was purchased (94.50 strike price – 94.21 EDM0 futures price). By being long the 94.50 call, the holder is entitled to assume a long position in the futures contract at the 94.50 strike price. In our example, buying this particular call meant sacrificing participation in the first .29 of the rally. Had a call option with a lower strike price been purchased, the profit potential from the hedge would have been greater, but the initial premium outlay would have been greater as well.

Holding the option position from March 1 through April 15 cost the holder only .015 in time premium (the initial .11 paid was all time premium, while the liquidation price of .175 consisted of .08 of intrinsic value, and .095 of recovered time value). Combining the .015 in lost time premium and .29 sacrificed by buying an out-of-the-money option, we get a total of .305. Multiplying by four contracts, the result is the $3,050 dollar difference between the actual profit ($650), and the total dollar value of the 37 basis point move on four futures contracts ($3,700).

Obviously, the final outcome in our example is only one out of many possible scenarios. For example, we assumed throughout that the hedge was not lifted or modified prior to April 15. Clearly any change in this assumption would significantly alter the results of the hedge. Our example clearly illustrates that getting market direction right is necessary, but not always sufficient for implementing a successful hedge. We have shown that the correct weighting and fine-tuning of the hedge (i.e., choosing the optimal strike price), and skillfully managing basis risk are also critically important.

Finally, one should remember that the most important decision for the risk manager is whether to hedge at all, knowing that with the benefit of hindsight, deciding to hedge will not always prove to have been the correct choice. However, once the decision to hedge has been made, risk managers with interest rate risk out one to two years will find that Mid-Curve options offer a unique tool for managing those positions. By providing low cost exposure to points on the yield curve where a considerable amount of interest rate risk is concentrated, Mid-Curves provide a highly efficient and flexible means of modifying a portfolio's risk profile, acting on expectations, or simply protecting against unwelcome surprises.

5-Year Eurodollar Bundle Options

by Ed Rombach

The Underlying Contract: The 5-year Bundle

The underlying for these options is the 5-year Eurodollar bundle, which is an equally weighted strip of the first twenty quarterly ED futures. Bundles have been traded at CME since September 1994 and, along with ED Packs, have become an extraordinarily popular tool for those who manage interest rate exposures out to ten years on the yield curve.

Contract Specifications

- CME 5-year Bundle options are listed in the March quarterly cycle, along with two serial expiration months. At any given time, two serial and two quarterly expirations are listed.
- Termination of trading is on the Friday prior to the third Wednesday of the expiration month, identical to that of Eurodollar serial and Mid-Curve options.
- 5-year bundle options are American style.
- Strike prices are listed in 25-basis-point-increments.
- 5-year bundle options also have characteristics that make them completely different from any other exchange-traded option.
- Strike prices are expressed as the average price of the contracts in the 5-year bundle.
- Bundle options expire into positions in the first twenty quarterly Eurodollar contracts.
- Premiums are quoted in terms of IMM index points, where one basis point (.01) = $500.

Bundles are traded on the basis of the average net change of the individual contracts relative to the previous day's settlement price (e.g., -2 bid/-1.50 ask). For option trading, bundle prices are also expressed as the average of the 20 individual futures prices that constitute the bundle (e.g., 94.4140). These average bundle prices are disseminated by CME via the existing quote vendor network just as if they were futures prices.

Two different 5-year bundles trade at any one time. One bundle has the first quarterly future as its lead contract, while the other uses the second quarterly future as its lead contract. For example, on February 25, 1998, there will be one bundle listed whose lead month is the March 1998 Eurodollar future, as well as a "lagged" bundle which uses the June 1998 contract as the lead month. Listing both bundles simultaneously allows CME to list two quarterly options (March and June in this case), as well as two serial month options (April and May) based on the June 5-year bundle.

Example

Figure 1 illustrates how the net change and average bundle prices are calculated, as well as how the at-the-money, and two closest out-of-the money strike prices are determined. The average 5-year bundle price in this example is 93.4140. This price is calculated by adding together the individual contract prices, and dividing the sum by twenty (the number of contracts in the 5-year bundle). The net change price is -2, which is equal to the sum of the net changes of each of the twenty contracts, then dividing by twenty.

Figure 1

ED Contract	Price	Net Change	Exercise P&L: 93.25 call	Average Bundle Price
Mar-98	93.965	0	$1,788	93.4140
Jun-98	93.845	0	$1.488	
Sep-98	93.75	0	$1,288	
Dec-98	93.63	0	$950	ATM Strike
				93.50
Mar-99	93.61	-1	$900	
Jun-99	93.56	-1	$775	Higher Strike
Sep-99	93.52	-1	$675	93.75
Dec-99	93.45	-1	$500	
				Lower Strike
Mar-00	93.45	-2	$500	93.25
Jun-00	93.41	-2	$400	
Sep-00	93.38	-2	$325	
Dec-00	93.31	-2	$150	
Mar-01	93.31	-3	$150	
Jun-01	93.27	-3	$50	
Sep-01	93.24	-3	($25)	
Dec-01	93.17	-3	($200)	
Mar-02	93.17	-4	($200)	
Jun-02	93.13	-4	($300)	
Sep-02	93.09	-4	($400)	
Dec-02	93.02	-4	($575)	
Avg. px	93.4140		$8200	
Net Change		-2.00		

The average bundle price of 93.4140 corresponds to a daily net change of -2 in the price of the bundle. CME trading conventions are such that bundles are quoted and traded on the net change basis, in quarter tick increments (e.g., -2.25 Bid/-2 Ask). A .25 change in the net change bundle price results in a .0025 change in the average price. In the above example, a decrease in the bundle's net change price from -2.00 to -2.25 would reduce the bundle's average price from 93.4140 to 93.4115; a net price change increase to -1.75 increases the average price to 93.4165. In dollar terms, both the .25 net price change and the .0025 average price change represent $125.

Strike Prices

Because the average bundle price in figure 1 is 93.4140, the initial at-the-money strike price is therefore 93.50. The next strike higher would be 93.75, the strike lower would be 93.25. Strike prices are added as necessary to keep this pattern in place as market prices change. For example, if the average price of the 5-year bundle rose to 93.75, the 93.75 strike would then be the at-the-money strike, and a 94.00 strike would be listed as the next upside out-of-the-money strike price.

Option Premiums

Bundle option prices are quoted in a manner similar to that of other CME interest rate options; the key difference is the dollar value of a .01 price increment. Because the size of a position in the 5-year bundle is twenty times that of a position in a single underlying futures contract, the dollar value of a .01 "tick" is $500 for a bundle option, versus $25 for an option on a single future.

Exercise

It is important to remember that Eurodollar bundle options are not cash settled. A trader exercising an option on the 5-year bundle receives individual futures positions in the front twenty Eurodollar contracts at the strike price. Figure 1 shows that based on the average bundle price of 93.4140, a trader exercising a 93.25 call is assigned long positions in all 20 futures contracts at a price of 93.25. The result is a profit of $8,200 per option (93.4140 - 93.25 x 20 contracts x $25 = $8,200). The dollar value of the positions assumed will be equal to the number of basis points the option is in the money, multiplied by $500 per basis point (16.4 basis points x $500 = $8,200).

Individual futures positions within the strip will show profits or losses, depending on the shape of the yield curve. The effect of yield curve shifts within the front twenty contracts on the option price will depend on whether the shift affects the overall average price of the bundle. For example, if the yield curve steepens in such a way that the price of the front ten contracts increases by 5 basis points, and the price of the remaining ten contracts falls by 5 basis points, the average price of the bundle will remain unchanged, as will the intrinsic value of the option.

Some Likely Applications

Options on 5-year Eurodollar bundles allow traders to buy or sell the right, but not the obligation, to buy or sell the underlying bundle. Conceptually this works the same way as option on futures. For example, if the average settlement price on a given day for the first twenty contracts in the Eurodollar futures strip was 93.96 / (6.04%), a standard strike price level of 94.00 (6.00%) for options on the 5-year bundle would be considered the at-the-money strike for both puts and calls just as with conventional options on individual futures. Similarly, strike prices at 94.25 and 93.75 would be just out-of-the-money for the calls and puts respectively. Market participants in the OTC swap derivatives market have commented that this development would be tremendously valuable for hedging and pricing swaptions. Certainly, at first glance a short-term option on a bundle looks an awful lot like a short-term option on a swap or swaption. The difference, though, is that a bundle comprises an even number of futures in each contract month of the term bundle. As such, a bundle is really much more like a synthetic zero coupon bond. It may not be exactly like comparing oranges to oranges, but maybe more like comparing oranges to tangelos.

Portfolio theory states that the volatility of an index will always be less than the weighted average sum of the volatilities of the component parts in the index. This notion may lead one to conclude that the volatility of a swap rate will be less than the volatilities of the component FRAs or futures in the case of a strip. However, such is not the case. In fact, swap rates calculated from historical FRA data will exhibit marginally higher volatility than the averaged sum volatilities of FRAs themselves. In other words, the whole is usually equal to more than the sum of its parts. This is true as long as the covariance between the FRA rates is positive.

For example, during the period from 9/15/97 to 12/15/97 the actual rate volatility of a 5-year forward swap starting at the Mar 98 futures expiration and maturing at the Mar 03 expiration (i.e., a 5-year, Mar 98 IMM swap), calculated from the futures prices and unadjusted for convexity was 11.59%. This compares with a volatility of 11.54% for an evenly weighted sum of each of the individual futures contracts during the same period of time. However, a much lower volatility of 9.68% can be derived if the volatility of each futures contract is weighted by the relevant PV factor in the strip. By contrast, the actual volatility of the 5-year bundle starting with the Mar 98 contract during the same time period was 11.33%.

Note that the price of the bundle is just a simple averaging of the twenty contracts, whereas the equivalent swap rate is determined from an internal rate of return calculation that places proportionately greater value on the component FRA rates at the front end of the curve. It is also useful to compare these volatilities with that of a highly correlated futures contract like the Mar 00 future, which could be seen as a proxy for either the IMM swap or the 5-year bundle as the correlation of yield changes between all three of these indices was in excess of 99.50% from 9/15/97 to 12/15/97. Moreover, there are short-term Mid-Curve options on the Mar 00 contract whose expiration will closely match the expiration of the 5-year bundle. In connection with this, it doesn't take much imagination to guess that market makers in options on bundles will look to Mid-Curve options as an efficient way to lay off their risk.

Some swap traders initially were critical of bundles because CME only recognizes trades that include the same number of futures contracts in each contract month, as opposed to an orthodox duration-weighted strip hedge, which is front-loaded and tapered at the tail end. For example, say a $100m, 5-year swap requires a strip hedge of 1,700 futures that is front loaded with 99 contracts in the front month and gradually declining to 73 contracts in the last month of the "Gold" pack.

Instead of executing this strip in the conventional way, at times it might be easier to approach the task by layering in a series of bundles consisting of a seventy-five-lot 5-year bundle; a five-lot 4-year bundle; a six-lot 3-year bundle followed by a six-lot 2-year bundle and a five-lot 1-year bundle. The resulting layer cake of bundles would be weighted at the front end and add up to the same number of futures as in the strip hedge, and very nearly replicate the risk profile of the orthodox strip hedge. Alternatively, the same task might be accomplished by stacking a series of packs consisting of 97 contracts in each of the front four White contracts, followed by 92 in each of the Reds, 86 in the Greens, 80 in the Blues and 75 contracts in each month of the Gold pack.

Conceptually, a similar layer cake can be constructed from options on the 5-year bundle combined with quarterly options on the front futures contract (as a proxy for the Whites), 1-year Mid-Curve options on the first contract in the "Red" strip (proxy for the Reds), and 2-year Mid-Curve options on the first contract in the "Green" strip (proxy for Greens, Blues, and Golds). Those market participants who are keen to construct orthodox duration-weighted hedges may want to pursue this approach, but it is not clear that the extra work involved will yield any better results than simply going ahead and hedging a swaption position with an option on a bundle by itself.

Based on market prices as of 12/15/97, a 3-month at-the-money option to receive fixed at 6.14% (semi-30/360) and pay floating on a 5-year swap priced at 13.60% implied volatility (European-style exercise) cost .651 of par or $651,000 on a $100m notional amount. The underlying IMM forward swap would have required a futures hedge of 1,670 contracts. If a slightly lower implied volatility of 13.35% is used to price an at-the-money call option on the 5-year bundle expiring at the same time as the swaption, the option would be worth 15.7bp valued at $655,475, (1,670 x 15.7 x $25/per bp). The extra value for the option on the bundle, despite the lower implied volatility, can be attributed to the American-style option pricing. By contrast, a layer cake option hedge approach might consist of 1,480 at-the-money call options on the 5-year bundle ($580,900); 120 at-the-money 2-year Mar 00 mid-curve calls at 18bp / $51,000 (15.40% imp vol); 48 at-the-money 1-year Mar 99 Mid-Curve calls at 18bp / $21,600 (15.50% imp vol); and 20 Mar 98 at-the-money quarterly calls priced at 9bp / $4,500 (7.97% imp vol) for a total of $658,000. Clearly, this hedge is not perfect but it goes a long way toward replicating the option-related risk profile of the swaption. All of these options expire at 2:00 p.m. Chicago time on the Friday before the third Wednesday of the month, with the exception of the Mar 98 quarterly options which expire on the Monday before the third Wednesday of the month.

How the layer cake option hedge performs, relative to just the bundle option as a stand-alone hedge, would depend largely on what happens to the yield curve. The bundle option would outperform the layer cake option hedge if the curve were to flatten. In a steepening curve environment, because the layer cake option hedge would have more optionality concentrated at the front end compared with the stand-alone bundle option hedge where the underlying strip of futures would be evenly distributed throughout the 5-year term, the bundle option would under-perform.

Monetizing the Convexity Bias

Swap traders who are short Eurodollar futures strips, either as a hedge against a fixed rate receiver swap or a net receiver position in a portfolio of swaps, are long convexity because when rates rise the futures will outperform the swap position and as rates fall the swap position will outperform the futures hedge. This advantage of being short the futures results in pricing of the futures strip at a yield premium to the equivalent term swap rate to account for the embedded optionality, otherwise known as the convexity bias.

Over time, if actual volatility is less than suggested by the bias, the hedged swap position described above will lose money as the convexity bias narrows. Conceptually it would be like buying an option straddle or strangle and seeing it expire at a lesser value. To realize the optionality of the convexity bias over time on a short futures/short swap position, one must sell additional futures as rates fall and buy back some of the short futures hedge as rates rise.

Based on market conditions at the expiration of the Dec 97 Eurodollar futures, a $100m, 5-year spot swap position would have required a duration-weighted hedge of 1,700 futures which would be weighted with 98 contracts in the first month and scaled down to 73 at the end of the Gold pack. Putting aside for the moment the passage of time which reduces the number of futures needed in the hedge, for every 25bp move up or down in rates, the hedge would require roughly twenty additional or fewer contracts.

Options on the 5-year bundle present a way to automatically adjust the size of the hedge and at the same time offset the erosion of time value in the bias for receiver swap/short futures hedged positions. Incremental one-lot amounts (20-contract strip) of at-the-money and out-of-the-money puts and calls on the 5-year bundle can be sold at specific strike prices geared to adjust the size of the hedge in a dynamic rate environment, like the way a thermostat adjusts the temperature in a house. If rates move up, the short puts would be exercised, thus reducing the size of the futures hedge. If rates move down, the short calls would be exercised, which would increase the number of short futures in the hedge. During that time, the risk manager hedging the swap position would try to selectively take in enough premium, quarter-by-quarter, by selling straddles or strangles to offset the expected time value decay in the hedged swap.

This technique can be thought of as a way to add convexity to the swap within the limitations of available strike prices that could be sold. Inefficiencies between the implied volatility of options on bundles and the volatility implied by the convexity bias for term swaps could be exploited to enhance the profitability on the warehousing of swap portfolios. This ability translates into greater certainty for carrying the swap at a profitable margin during a defined holding period.

Synthetic TED Spreads

Arbitrage desks have made a business in recent years out of spreading Eurodollar futures strips against 2-year and 5-year T-note futures, as well as against cash-on-the-run Treasury notes. However, recently these markets have become so efficient that much of the incentive has been arbitraged away. The introduction of options on bundles will make it possible to exploit differences in the implied volatility between these markets in order to create an edge in buying or selling TED spreads.

First, take into account that implied volatility of options on Treasuries are expressed in terms of price volatility and options on Eurodollar futures are expressed in terms of rate volatility. Say implied volatility for options on the 5-year bundle appears to be rich relative to options on the 5-year T-note future, and you judge the 5-year TED spread (5-year Eurodollar futures strip minus 5-year cheapest to deliver) to be wide, so you decide to sell it. One way to approach the trade would be to buy a put option on the 5-year T-note future and sell a put on the 5-year Bundle, optimally at a net credit. If rates go down, both options should expire worthless and you keep the credit. If rates go up, both options go into the money; if the TED spread narrows you should make more money on the long 5-year T-note put than you lose on the short put on the 5-year Bundle.

Product Evolution

Quarterly options on the 5-year Bundle appear to represent an invigorated re-commitment on the part of the Exchange to meet the needs of its members and the global trading community. Assuming this new hybrid derivative is well received, it is likely to be only the beginning of a whole new family of variations on the same theme. Ultimately, tailored options on odd date bundles and packs is the direction toward which CME must go if the Exchange wants to attract a meaningful flow of hedge-related business from the swaption market.

There are many more possible swaption structures, driven in large part by the origination of the voluminous callable U.S. Agency debt issued by borrowers like Federal Home Loan Bank (FHLB), Student Loan Mortgage Association (SLMA), Federal Farm Credit Bank (FFCB), Federal National Mortgage Association (FNMA) and the Federal Home Loan Mortgage Corporation (FHLMC). These quasi-governmental borrowers issue callable debt with final maturity dates from one year on out to twenty years that have initial call dates ranging from three months on out to five years. In this context, the possibilities for options on bundles appear endless, and CME and its members stand to garner a meaningful chunk of the related order flow from the swap derivatives transactions that are tied to this business.

Ed Rombach is a risk management consultant and principal of ED-ROM, L/C Telephone: (781) 639-0723, or email: club-ed@attbi.com

For further information on CME Interest Rate futures and options, please call 1800-331-3332 or visit the CME Web site at www.cme.com.

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