Is liquidity self-fulfilling?*

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September 23, 2002

Abstract

(see next page for abstract)

*Acknowledgements: this research forms part of the European Commission TMR network project on "Financial Market Efficiency and Economic Efficiency," grant contract no. FMRX-CT96-0054. I thank Julian Franks and a referee for detailed comments on the previous draft, and Margaret Bray and Thierry Foucault for discussions on an earlier version of this paper.

ABSTRACT: A security market where the relative incidence of informed and uninformed trading determines liquidity may have more than one equilibrium. Equilibrium with high liquidity has a low bid-ask spread. This increases participation by traders who want to hedge risk exposure, as opposed to trading on private information, and justifies the small price impact of trades. Equilibrium with low liquidity has a high bid-ask spread. This deters some hedgers, increasing the relative incidence of informed trading, which justifies the larger spread. This analysis casts doubt on the relevance of comparative statics results in the existing literature relying on exogenous liquidity traders.

Journal of Economic Literature classification number: G14. Keywords: Liquidity, Asymmetric information, Informed trading, Securities prices, Price formation, Market-maker.

1 Introduction

Research on the microstructure of security markets has concluded that the relative incidence of informed *versus* uninformed traders is a crucial determinant of liquidity (see, e.g. Spatt (1991)). If there is a relatively low volume of uninformed trade a given volume of buy orders in the market will cause the price to rise by a large amount (Kyle (1985)). If a market-maker quotes bid and ask prices for a given order size, a greater incidence of informed trade will cause a wider bid-ask spread (Glosten and Milgrom (1985)). Empirical studies have confirmed that informational asymmetry seems to be an important component of bid-ask spreads and other aspects of market-maker behaviour (Stoll (1989), Madhavan and Smidt (1991) and Bhattacharya and Spiegel (1998)).

From the viewpoint of an uninformed trader, the finite depth caused by informed traders is like a transaction cost: if he buys an asset then resells immediately, he will lose money because of the lower price on resale. Hence, the presence of informed traders should deter participation by uninformed traders. This can be verified by endogenising uninformed agents' trading decisions by modelling them as hedgers with an initial risk exposure that is correlated with the asset payoff. This is a standard device to represent the motives of uninformed traders, as in Spiegel and Subrahmanyam (1992), following the earlier work in market microstructure that did not model the motives of uninformed traders and simply assumed they traded an exogenous random amount. Uninformed agents then face a trade-off: the more they hedge to reduce their risk exposure, the more money they lose through market illiquidity. This costly illiquidity results from the presence of informed traders, so that, in equilibrium, the hedgers' monetary loss is the informed traders' gain.

Consider a securities market where a market-maker quotes bid and ask prices for a given order size. In equilibrium, then, the relative incidence of trading by informed traders *versus* uninformed traders will cause a bid-ask spread to emerge, and the trading decisions of uninformed traders (as well as informed traders) will be optimal responses to this bid-ask spread. Will liquidity be high or low in equilibrium? Suppose first that the bid-ask spread is wide. This will tend to reduce trading by hedgers (clearly, for some specifications of the model it could also reduce trading by informed traders, but I will focus on situations where the effect is strongest on uninformed agents). But this lower incidence of uninformed trading, relative to informed trading, will in turn justify the wide bid-ask spread. Hence, an illiquid equilibrium seems a plausible outcome. Now suppose, on the other hand, that the bid-ask spread is small. This could encourage hedging, reduce the relative incidence of informed trading and hence justify a small bid-ask spread. Intuitively, it seems plausible that multiple equilibria could occur, and I show that they can. I present a model with simple functional forms, based on the Glosten and Milgrom (1985) model of price-setting, and show that for a range of parameter values it will have two equilibria.

This result raises the question of robustness with respect to the price-formation mechanism (and functional forms for utility functions and random variables). The intuition underlying the result should work when the cost of illiquidity is manifested in price impact instead of a bid-ask spread. Apart from the Glosten and Milgrom (1985) model, Kyle's (1985) model has become a canonical framework for representing stock price formation, featuring a market-maker who sets price in response to aggregate order flow, with CARA preferences for the traders and jointly normally distributed random variables. Spiegel and Subrahmanyam (1992) modify Kyle's model to include rational uninformed agents, by giving the uninformed a hedging motive, and show it has a unique linear equilibrium.[1] However, although those particular functional forms in that model may imply uniqueness, there is nothing pathological about the existence of multiple linear equilibria and in subsection 4.4 I briefly describes a variant of the model of Spiegel and Subrahmanyam (1992) that also has two equilibria. Hence, my conclusion is that non-uniqueness is a robust possibility when uninformed agents are modelled as rational hedgers, in the sense that it can occur with models in the style of either Kyle (1985) or Glosten and Milgrom (1985).

Many economic models, including models of securities markets, can have multiple equilibria. Several papers have described multiplicity of equilibrium in financial markets (including Admati and Pfleiderer (1988), Pagano (1989a), Pagano (1989b), Bhattacharya and Spiegel (1991), Saint-Paul (1992), Gale (1992), Pagano (1993), Rochet and Vila (1994) and Bhattacharya, Reny and Spiegel (1995)). Section 2 of the paper reviews related literature and explains the similarities and differences between the multiplicity studied here and the multiplicity in those papers.

[1] Although non-linear equilibria in that model have not been ruled out, there seems to be a presumption among many researchers that they do not exist. I discuss below work by Bhattacharya and Spiegel (1991) and Rochet and Vila (1994) on non-linear equilibria in other related models.

Understanding liquidity is an important priority for research on financial markets. For example, in order to have a good understanding of the important phenomenon of financial innovation we need to understand how liquidity is created and maintained. Research on the introduction of futures contracts is one instance of this (Working (1953), Gray (1970), Sandor (1973), Hieronymus (1977), Cornell (1981), Silber (1991), Cuny (1993)). As several of these authors note, many futures contracts have low liquidity even though their fundamental value as hedging instruments is superior to successful contracts (see, for example, Working's (1953) discussion of the North Pacific Coast wheat futures, and the overview given in the introduction to Cuny (1993).) The analysis presented in this paper, using the standard models of how liquidity is determined, can explain how liquidity can fail to establish itself. On the other hand, one cannot regard this as a complete explanation for such phenomena. Among the interesting problems that remain open for future research into liquidity formation is the need to develop dynamic models of liquidity formation that explain how liquidity is formed over time and how it interacts with liquidity in other markets.

The analysis of this paper also has a more negative implication however. There is an extensive literature on market microstructure, mostly based on exogenous liquidity traders, and containing predictions based on the comparative statics properties of the models (see O'Hara (1995) for an overview of this literature). This paper calls such predictions into question because it suggests that, if the uninformed trading motive is endogenised, such models may have more than one equilibrium. In that case comparative statics cannot be relied on because, comparing two different markets (or one market at two different times), the two markets may differ because they are in different equilibria, not because they are in the same equilibrium under different values of exogenous parameters.

The organisation of the rest of the paper is as follows: section 2 reviews related literature. Section 3 presents the model. In section 4 I compute equilibrium and show that multiple equilibria will occur for a range of parameter values. One equilibrium has high trading volume and liquidity, the other has low volume and liquidity. An extension (subsection 4.3) shows that with more heterogenity among hedgers, the market can have more than just two equilibria. I show that the more types of hedger there are, the more equilibria can occur. I also (subsection 4.4) briefly discuss robustness with respect to the trading mechanism and the functional forms used. Section 5 concludes.

2 Related literature

The ideas explored in this paper are very simple, but the main point has not been addressed in the existing literature - i.e. that endogenising the motives for uninformed trade in the presence of informed traders enhances the possibilities of multiple equilibria with different degrees of liquidity. However, the possibility of this kind of multiplicity of equilibrium seems important. One related paper is Admati and Pfleiderer (1988) (similar ideas are also discussed in Kyle (1984)). To rationalize the existence of seemingly inexplicable variations in trading volume across the day, they explored a multi-period version of Kyle's (1985) model. As in Kyle (1985), the uninformed agents' trade is an exogenous random amount, but some of them have the flexibility to decide the timing of their trade. Thus, the motivation of the uninformed traders is partially, but not completely endogenised. In equilibrium these uninformed traders will choose to cluster together so that some periods will be liquid and others illiquid.

Some papers do explore multiple equilibria, but model illiquidity as related to risk, not asymmetric information. In Pagano (1989a), a low-liquidity equilibrium is one where few investors participate in the market. Since the market then has low risk-absorption capacity, prices are volatile and this justifies investors' decisions not to participate. As Pagano stresses, the result depends crucially on transaction costs (in the form of a lump sum cost of participating): otherwise the principle of local risk neutrality would imply that non-participating investors would be willing to accept small (non-zero) positions in return for large risk premia. Arguably, global financial markets do contain many investors who would easily be able to enter any securities market in an economically-developed country in order to collect a premium return in exchange for a small amount of diversifiable risk, so this limits the applicability of the analysis. Pagano (1989b) is a model of competing markets for the same security where agents face a participation constraint that allows them to trade in only one of the two markets. There may be multiple equilibria where liquidity is concentrated in one market or the other. In more recent work, Pagano (1993) presents a model of entrepreneurial share issuance in which the presence of financial market imperfections (either transaction costs of holding equities, or credit market imperfections) causes multiple equilibria; in an equilibrium with low liquidity, few companies choose to float on the stock market. Saint-Paul (1992) also presents a model with multiple equilibria where, in an equilibrium with low stock market participation and lump-sum costs of participation, firms' inability to use the stock market to diversify causes them to choose inferior but less risky production techniques. Also, Gale (1992) studies security design in a risk-sharing model where each security has a fixed cost of being issued and shows that multiple equilibria will occur.[2]

Like this paper, Pagano (1989a), Pagano (1989b), Pagano (1993), Saint-Paul (1992) and Gale (1992) are based on the idea of coordination failure, or multiple equilibria with different degrees of liquidity. The model presented in this paper is different from the above work in three respects: first, the results do not depend crucially on transaction costs. They can be derived in the absence of any transaction costs. As Saint-Paul (1992) notes (footnote 2), non-convex transaction costs such as lump-sum participation costs are a powerful way of generating strategic complementarity and multiple equilibria, which are harder to derive in the absence of such costs. Arguably at least, the transaction costs of financial market participation are small. Second, the results do not depend on participation constraints that limit the number of markets an agent can trade in (in contrast to some, but not all of the above papers). Third, the economic

[2] Note also Cuny (1993), Williamson (1994), and Allen and Gale (1994), which do not feature multiple equilibria, but do use participation constraints or transactions costs to explain liquidity in securities markets.

reason for illiquidity is different. In the analysis presented here, illiquidity is the result of informational asymmetries. In the extreme, the effects studied here would resemble the stock exchange where "there is no trading there, because all the trading is insider trading."

Many papers within the microstructure literature have looked at a specific equilibrium without completely ruling out the possibility of other equilibria within the model. For example, Kyle (1985) looks only at the set of linear equilibria, within which there is a unique equilibrium. A few papers have gone beyond this to investigate the possibility of non-linear equilibria and to characterise the full set of equilibria. Bhattacharya and Spiegel (1991) is a complete analysis of the full set of equilibria in a model where a single informed trader interacts with a group of competitive uninformed agents (outsiders) via a Walrasian auctioneer. The uninformed agents are modelled as rational traders with a hedging motive, not exogenous liquidity traders. In their model, there is a single linear equilibrium but many non-linear equilibria. They show, however, that the equilibria are all the same in terms of the residual risk borne by the outsiders and in the informational content of prices.[3]

Rochet and Vila (1994) address the question of multiple equilibria in market-maker models with exogenous noise traders in the style of Kyle (1985). The most relevant part of their paper for the issues discussed here is the example

[3] The main focus of their article is to investigate how these equilibria collapse as a function of the exogenous parameters when the informational motive is too strong relative to the hedging motive. See further discussion of their paper below. Bhattacharya, Reny and Spiegel (1995), another paper focused on characterising conditions when the equilibrium collapses, also treats the whole set of equilibria and not just the linear equilibria.

they present in appendix E, showing that the Kyle (1985) model, modified with different functional forms for random variables, can have multiple equilibria. However, as in the earlier paper of Bhattacharya and Spiegel (1991), the multiplicity in their example does not entail different levels of liquidity in equilibrium: the informativeness of prices is constant across equilibria. In the main part of their paper they are able to show uniqueness for a variant of the Kyle (1985) model in which agents can submit limit orders, although that result is of limited significance for the present analysis since it relies on exogenous liquidity and does not model the decisions of uninformed traders.

A number of papers study market breakdown whereby, in a variety of models, the informed trading motive overwhelms the uninformed trading motive to the point that equilibrium with trading ceases to exist (Glosten (1985), Bhattacharya and Spiegel (1991), Hellwig (1992), Bhattacharya, Reny and Spiegel (1995). There are clear similarities between such breakdowns and the results studied in this paper. The reason for breakdown or illiquidity is the same: uninformed agents are driven out of the market because of their reluctance to engage in trade with better informed agents. However, there are two differences. First, the market breakdown papers vary exogenous parameters and see how the equilibrium is affected, while I study multiple equilibria for a given set of parameters. Secondly, in this paper I study equilibria with different degrees of liquidity but all with positive trading volume, rather than complete breakdown.

3 The Model

I consider a model with a single risky security and two types of agents, hedgers and speculators. Hedgers are risk-averse and have risky endowments that are correlated with the security, while speculators have private information on the value of the security (modelling uninformed trade as motivated by a desire to hedge risky endowments is a standard device introduced by Hellwig (1980) and Diamond and Verrecchia (1981) in REE models, and Glosten (1989) and Spiegel and Subrahmanyam (1992) in market-maker models). The price is set by a market-maker as in Glosten and Milgrom (1985).

The security payoff, denoted, is equally likely to be either 1 or -1. (There is no loss of generality in setting the mean to zero: one could add on a constant to the mean, to the expression for the security price, and to agents' endowments.)

Hedgers have a concave utility function and a risky endowment of $\pm\widetilde{x}$ (equiprobable). They first find out whether their endowment is $\widetilde{x}$ or $-\widetilde{x}$, then they are able to trade, and then the security payoff $\widetilde{x}$ is realized. If the endowment is $\widetilde{x}$, they can hedge their endowment risk by selling short one unit of the security (and conversely if their endowment is $-\widetilde{x}$ they can hedge with a long position). There are two types of hedgers with different degrees of risk-aversion; their von Neumann-Morgenstern utility is given by $U_i(w) = w$ for $w \leq 0$, and $U_i(w) = r_i w$ for $w > 0$, where $i = H$ or L and $0 < r_H < r_L < 1$. (Here "H" stands for high risk aversion and "L" for low, but note that higher risk-aversion corresponds to a lower value of r_i.) From now on I shall write the utility in the more compact notation

$$U_i(w) = \min\{w, r_i w\}. \tag{1}$$

For technical reasons each type of hedger should be considered as a unit mass of a continuum of infinitesimal agents with identical endowments.

Speculators, or informed traders, know the realization of $\widetilde{x}$ in advance of the trading round. For simplicity they are assumed risk-neutral (actually, this makes no difference in equilibrium as they know the asset value perfectly and hence do not bear any risk).

It will be convenient to think of a given order as a sample from a large population of hedgers and speculators. The frequency of speculators is denoted π_s and the frequency of hedgers is $(1 - \pi_s)$. Furthermore, the frequency of the hedgers with high risk-aversion is π_H, and that of the hedgers with low risk-aversion is π_L. (Thus $p_s + p_H + p_L = 1$). Prices are set by a risk-neutral market-maker who observes one order at a time, infers (in equilibrium) the relative likelihood that the order comes from an informed trader or a hedger, sets the price equal to the conditional expected value of the security, and meets the order from inventory.

4 Liquidity in Equilibrium

The equilibrium bid-ask spread will depend on the relative frequency of trades from informed and uninformed agents (i.e. respectively speculators and hedgers). The speculators, since they know the asset value, will always trade so long as the price does not fully reflect this information. On the other hand the hedgers must balance the benefits of trading to hedge their initial risk exposure against the cost implied by the bid-ask spread. Depending on the size of the bid-ask spread and on their own degree of risk-aversion, they may or not may not choose to trade.

The hedgers will trade a positive or negative amount (depending on their endowment shock) that I denote t. This quantity t will be derived below; it will also be verified below that the optimal t is the same regardless of the degree of risk aversion (i.e. it is the same for U_L and U_H). The speculators will also choose t since any other quantity would reveal their information to the market-maker.

To derive the equilibrium, let us first suppose that all agents trade in equilibrium. Let p denote the price set by the market-maker in response to a buy order (the price in response to a sale is then $-p$ and the bid-ask spread is $2p$). Since all agents are active, the market-maker's belief that an order comes from an informed trader is π_s. Therefore the security price equals this probability, since if an informed trader buys the asset is worth 1, while if an uninformed trader buys the expected value of the asset remains 0.

4.1 Hedger Behaviour

Now consider the hedgers' behaviour. The hedgers' initial endowment is $\pm\widetilde{x}$. For the case of a positive hedging need (initial endowment $-\widetilde{x}$), a hedger who buys t units of the security at price p has wealth

$$t(\widetilde{x} - p) - \widetilde{x}. \tag{2}$$

Depending on the realized value of $\widetilde{x}$, this can take two possible values:

Case (i): $\widetilde{x} = 1$

Wealth: $t(1-p) - 1$

Utility: $\min\{[t(1-p) - 1], r_i [t(1-p) - 1]\}$.

Case (ii): $\widetilde{x} = -1$

Wealth: $t(-1-p) + 1$

Utility: $\min\{[1 - t(1+p)], r_i [1 - t(1+p)]\}$.

The expected utility of the hedger is:

$$\frac{1}{2}[\min\{t(1-p) - 1, r_i(t(1-p) - 1)\} + \min\{1 - t(1+p), r_i(1 - t(1+p))\}]. \tag{3}$$

Note that in case *(i)*, wealth is positive if $t \geq \frac{1}{1-p}$ while in case *(ii)*, it is positive if $t \leq \frac{1}{1+p}$. Hence expected utility as a function of t is piecewise linear with kinks at $t = \frac{1}{1-p}$ and $t = \frac{1}{1+p}$. On the interval $[0, \frac{1}{1+p}]$ the derivative is

$$\frac{1}{2}[(1-p) - r_i(1+p)] \tag{4}$$

which is positive if, and only if, $r_i \leq \frac{1-p}{1+p}$. For t beyond this interval, the derivative is always negative: it is $-p$ on $[\frac{1}{1+p}, \frac{1}{1-p}]$ and $\frac{1}{2}\left[(r_i - 1) - p(r_i + 1)\right]$ for $t > \frac{1}{1-p}$.

It follows that uninformed agents of type i will choose to hedge when

$$r_i \leq \frac{1-p}{1+p} \tag{5}$$

in which case, they will trade

$$t = \frac{1}{1+p} \tag{6}$$

and obtain expected utility

$$\frac{1}{2}\left[\frac{1-p}{1+p} - 1\right] = \frac{-p}{1+p}. \tag{7}$$

4.2 Equilibrium Characterisation

To complete the derivation of the equilibrium we need to verify the condition that both types of hedgers choose to trade with the bid-ask spread set at $2\pi_s$ (i.e. $p = \pi_s$):

$$r_L \leq \frac{1-\pi_S}{1+\pi_S}. \tag{8}$$

This is a condition on the exogenous parameters of the model. So long as (8) holds the model will have an equilibrium with relatively high liquidity, i.e. where all agents trade and where the bid-ask spread is small.

The next task is to explore the possibility of equilibria with lower liquidity.

Let us now suppose that in equilibrium, only the hedgers with high risk-aversion ($r_i = r_H$) trade. The price set by the market-maker will be based on the inference that a given trade has a probability

$$\frac{\pi_S}{\pi_S + \pi_H} \tag{9}$$

of originating from an informed trader. Hence this will be the price set in response to a buy order and the bid-ask spread will be $\frac{2\pi_S}{\pi_S+\pi_H}$. The above derivations of hedgers' optimal demands for the asset imply that this equilibrium will occur when

$$r_H \leq \frac{\pi_H}{\pi_H + 2\pi_S} \leq r_L. \tag{10}$$

This condition guarantees that the more risk-averse hedgers choose to trade, while the less risk-averse hedgers decide that the benefit of hedging is outweighed by the cost of the bid-ask spread.

For some values of the exogenous parameters (r_H, r_L, π_H, π_L and π_s) the model will have only one of the two types of equilibrium. For example, if we make r_H close enough to r_L while holding the π's fixed, it can be seen that (10) will fail to hold. On the other had if r_L is too big condition (8) will fail.[4]

We can now show that there may exist two equilibria with different degrees of liquidity:

Proposition 1 (Endogenous Liquidity) *For an open set of values of the ex-*

[4]Indeed, if both r_H and r_L are too large in relation to π_s the model will fail to have any non-trivial equilibrium with strictly positive trading volume. However, this paper focuses exclusively on equilibria with strictly positive amounts of both informed and uninformed trade.

ogenous parameters there are two equilibria of the model. In one equilibrium, there is higher liquidity (smaller bid-ask spread) and all agents trade. In the other equilibrium, there is lower liquidity and only the more risk-averse hedgers and the informed agents trade.

Proof. We need to show that both conditions (8) and (10) may be satisfied simultaneously with strict inequality (if the conditions are satisfied with strict inequality, they will continue to hold if the parameters are perturbed and hence the result will hold on an open set). This is clearly possible by choosing r_H to satisfy

$$r_H < \frac{\pi_H}{\pi_H + 2\pi_S} \tag{11}$$

and r_L to satisfy

$$\frac{\pi_H}{\pi_H + 2\pi_S} < r_L < \frac{1 - \pi_S}{1 + \pi_S} \tag{12}$$

which is always possible so long as

$$\frac{\pi_H}{\pi_H + 2\pi_S} < \frac{1 - \pi_S}{1 + \pi_S}. \tag{13}$$

It is straightforward to verify that (13) always holds. ■

It is also immediate that:

Proposition 2 (Pareto Dominance) *The equilibrium with high liquidity Pareto-dominates the equilibrium with low liquidity.*

4.3 Extension to many types of trader

The result presented so far shows that with two types of hedger, there may be two equilibria: one where both types trade and one where only the more risk-averse types trade. The model can be extended to many types of uninformed trader, with different levels of risk aversion. There can be correspondingly many equilibria.

Assume there are n types of hedger indexed by $i = 1, ..., n$, each with utility function

$$U_i(w) = \min\{w, r_i w\} \tag{14}$$

and with frequency π_i (with $\pi_S = 1 - \sum_{i=1}^{n} \pi_i$). The types are ordered so that higher index numbers correspond to higher risk aversion (lower r_i):

$$r_n < ... < r_2 < r_1. \tag{15}$$

The specification of endowments and the other aspects of the model are as before. As previously derived, hedgers of type i will trade if

$$r_i \leq \frac{1-p}{1+p} \tag{16}$$

(equivalently, $p \leq \frac{1-r_i}{1+r_i}$). It follows that if hedgers of type i are willing to trade, so are more risk-averse hedgers, *i.e.* hedgers of all types $j > i$. Thus, in a given equilibrium there is a type $k \in \{1, ..., n\}$ such that hedgers of type $j \geq k$ trade, while types $j < k$ do not trade. Then the updating rule for the marketmaker's beliefs implies that the price is given by

$$p = \frac{\pi_S}{\pi_S + \sum_{i=k}^{n} \pi_i}. \tag{17}$$

Proposition 3 *For an open set of values of the exogenous parameters there are n equilibria of the model. In the k'th equilibrium (for $k = 1, ..., n$), type k trades and so do all the hedgers who are more risk averse than type k (i.e. types $i = k, ..., n$), while hedgers who are less risk averse than type k (types $i = 1, ..., k-1$) do not trade. Liquidity is decreasing in k.*

Proof. Fix π_s and $\pi_1, ..., \pi_n$ (all strictly positive). Define p_k to be the price that would prevail if hedgers of types k and higher were to trade, and types below k were not to trade. Thus

$$p_k = \frac{\pi_S}{\pi_S + \sum_{i=k}^{n} \pi_i} \tag{18}$$

Now define $\phi_k = \frac{1-p_k}{1+p_k}$. Note that $0 < p_1 < p_2 ... < p_n < 1$ and that $f : [0,1] \longrightarrow [0,1]$ defined by $f(x) = \frac{1-x}{1+x}$ is a strictly decreasing function, hence $0 < \phi_n < ... < \phi_1 < 1$. Also note that in the above construction, the ϕ's are continuous in the π's. Now choose $r_1, ..., r_n$ so that $0 < r_n < \phi_n < ... r_2 < \phi_2 < r_1 < \phi_1 < 1$. By construction this set of parameters has an equilibrium where hedgers of types k and above trade, and types below k do not trade, for each $k = 1, ..., n$. This property also remains if the parameters $\pi_s, \pi_1, ..., \pi_n, r_1, ... r_n$ are perturbed. ■

As before, it is immediate that the equilibria are Pareto ranked, with more liquid equilibria preferred. From the proof, it can easily be seen that for some other parameter values, the model will have fewer than n equilibria.

The extension in this section represents only one way to extend the model. One can envisage other extensions. For example, the uninformed agents in this model have a single kink in their utility function, implying that they will be either in or out of the market (this functional form was chosen because it is the simplest form possible to make the point). If they had two kinks, there would be the possibility that in one equilibrium they would trade a large amount and the market would be liquid, while in the other equilibrium they would trade less and the market would be less liquid.[5]

4.4 Robustness to model specification

The model presented here, although similar to standard models in the literature, is a simple and special one. Clearly, to the extent that multiplicity is possible with this model it would also be possible with more complex variants of this model, and it therefore seems preferable to focus on obtaining the result in the simplest possible framework. On the other hand it is important to know whether the results are robust with respect to features such as risk preferences, distributions of random variables, and price-setting mechanism. So, in a previ-

[5] The computation of equilibrium in that model would be more complex because trades could be of two different sizes. See Easley and O'Hara (1987) for computation of equilibrium in such a model, but with exogenous uniformed trades of different sizes. Informed traders are then able to decide which order size they wish to place, and in equilibrium different order sizes have different bid-ask spreads. Roell (1987) extends the analysis to a continuous distribution of (exogenous) uninformed trade sizes.

ous version of this paper (available on request from the author) I investigated a model with exponential (CARA) utility, normally distributed random variables, and a Kyle (1985) market-maker. I showed that, with suitably small transaction costs, this model also has multiple equilibria on an open set of exogenous parameters.

5 Conclusion

In this paper, I have used a standard model of security price formation where informed traders interact with uninformed traders to show that multiple equilibria may occur. In the equilibrium with high liquidity, trades have a small price impact (in the form of a small bid-ask spread) and all agents participate actively in the market. In the equilibrium with low liquidity, agents face a higher bid-ask spread: some hedgers (uninformed agents) drop out of trading and those that do remain trade less. Informed traders are also worse off.

This result can be given both a negative and a positive interpretation. Viewed negatively, it raises grounds for serious concern about the robustness of the predictions that have been generated by the large literature on market microstructure (see O'Hara (1995) for a comprehensive survey of this literature). In this literature, comparative statics results have been used as a basis for policy recommendations. This literature has relied on models with exogenous noise traders, used as a shorthand device to represent traders with non-informational motives for trade (such as the hedging motive used here.) Yet, I have shown that explicitly modelling those motives may lead to multiple equilibria, which calls the validity of comparative statics into question.[6]

On the other hand, viewed positively, the possibility of multiple equilibria (as studied in this paper) may help to improve our understanding of how liquidity is formed. In particular it is compatible with evidence from futures market innovations (see Cuny (1993)) that new securities may fail to develop liquid markets even though their fundamental economic characteristics appear sound.

[6] On the other hand, a model may have a unique equilibrium for some parameter values (see propositions 1 and 3 above). Given the specification of the model, and given those parameter restrictions, comparative statics analysis is valid. For example, Dow (1998) uses a similar model to explore liquidity links across markets for different securities. Although that paper does not discuss the possibility of multiple equilibria, it can be verified that the comparative statics results presented in that paper do hold for values of the exogenous parameters with a unique equilibrium, even though for other values of the exogenous parameters the model will have multiple equilibria.

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