The L-Stable Distribution and Firm-Size Related Momentum Effect

Abstract

Our analysis indicates that the fat-tails of stock returns are better described by an L-stable distribution. They therefore often fluctuate less than a Gaussian distribution would suggest. The fact that a stable but not normal distribution does not have well defined second and higher moments makes the use of asset pricing models inadequate. Our findings also suggest that return persistence varies with company size. Thus the momentum effect, an investment behavior related to return persistence, varies with firm size. A possible explanation of the firm-size related momentum effect is the existence of familiarity and herding in investing.

I. Introduction

Many studies note that returns on financial assets have stable distributions, which do not have finite second or higher moments [Mandelbrot (1960, 1963a, 1963b), Fama (1965), Fama and Roll (1968, 1971), Teichmoeller (1971), Officer (1972), Fieletz and Smith (1972), Leitch and Paulson (1975), Koutrouvelis (1980), Akgiray and Lamoureux (1989)]. If the distribution of a time series is stable but not normal, the second order moment is not well defined. Under this circumstance, the use of variance and beta as measures of risk becomes less meaningful.

Since its introduction in 1990 by Jegadeesh, the momentum effect has attracted the attention of many financial economists. The fact that the phenomenon cannot be properly explained by asset pricing theories leads to an intensive scrutiny of the sources of this effect. Persistence in stock returns, firm-specific and industry-specific explanations have been suggested as the causes of the puzzling phenomenon. Investment psychology has also been mentioned as a possible reason for the seemingly irrational market behavior.

As shown below, many studies have suggested that stock returns are best described by an L-stable-distribution. However, we are unaware of any study which has linked L-stable stock return distributions to the momentum effect. The purpose of this paper is to examine the statistical traits and psychological causes of this interesting market pattern. The organization of this paper is as follows. A literature review is given in Section II. Section III reviews the methodology of the two-step regression algorithm developed by Koutrouvelis. Data and findings are presented in Section IV. In the final section, we conclude that the momentum effect is caused by return persistence and investment psychology.

II. Literature Review

Discovery of the stable distribution is generally attributed to Levy (1937), and thus the name L-stable has been assigned to this distribution by Mandelbrot (1960, 1963b). Levy's study investigated the distribution of incomes and found distributions with much fatter tails than would be suggested by a Gaussian distribution. Although well known, Levy's work did not receive much attention in the finance literature until Mandelbrot published his study.

An L-Stable distribution is described by its characteristic function:

$$(1) \qquad \varphi(t) = \exp\{i\delta t - |ct|^{\alpha}[1 + i\beta \operatorname{sgn}(t)\,\omega(t,\alpha)]\},$$

where $\omega(t,\alpha)$ is $\tan(\pi\alpha/2)$ for $\alpha \neq 1$, and $(2/\pi)\log|t|$ for $\alpha = 1$; $0 < \alpha \leq 2$ is the characteristic exponent (a measure of fatness of the tails, with smaller α associated with fatter tails); $c > 0$ is the scale parameter (a measure of variability); δ is a location parameter (an indicator of the domain of attraction); and $-1 \leq \beta \leq 1$ measures asymmetry (i.e., symmetric if $\beta = 0$, asymmetric otherwise).

It is worth noting that the stable distribution has some interesting theoretical properties. When a distribution has an $\alpha \leq 1$, its mean is not defined; when $1 < \alpha < 2$, the mean of this distribution is finite but the second moment is theoretically infinite; when $\alpha = 2$, the distribution becomes normal with mean δ and variance $2c^2$. In addition, a stable distribution has the stability property that it forms a domain of attraction. These properties can be used for mathematical modeling and statistical analysis.

In addition to Mandelbrot (1960, 1963a, 1963b, 1982), important theoretical work on the L-Stable distribution was also undertaken by Gnedenko and Kolmogorov (1954), Feller (1971), Zolotarev (1986) and by Samorodnitsky and Taqqu (1994). The work by Mandelbrot and Feller laid the foundation for a number of important advances in this field of study.

Fama (1965), Fama and Roll (1968, 1971), developed the fractile method to estimate the parameters of a symmetric stable distribution. They also published probability tables for sample symmetric stable distributions. Their use of truncated means and their assumption of symmetry significantly reduce the computational requirements for estimating the parameters of a symmetric stable distribution. Given their assumptions, the procedure works quite well.

Since the seminal work of Fama and Roll (1971), other methods for testing the existence of the stable distribution have been proposed. Press (1972) used the method of moments to estimate the parameters of the L-stable distribution. Although this procedure can be used to investigate both symmetric and non-symmetric distributions, the amplitude of the sample characteristic function can overwhelm the harmonics, and thus results in inefficient estimation of both symmetry and location. Paulson, Holcomb and Leitch (1975) were the first to recognize the amplitude problem of the sample characteristic function impacting upon the estimation of symmetry and location. The authors standardized their data to overcome this problem and use a 20-point Hermitian quadrature to numerically implement their distance procedure. They, however, suggested that their procedure was less accurate for non-symmetric distributions. When they apply their procedure, one must also realize that the optimal degree of the Hermitian quadrature for a given series of stock returns is not definitive. Leitch and Paulson (1975) estimated the stable law parameters of stock prices. The authors used an alternative distance minimization procedure with numerical estimations by a 20 point Hermitian quadrature. Although it was not clear how issues of sampling and the degree of the quadrature impact estimated results, their study of stock price distributions suggests that "symmetry is definitely the exception and not the rule" (p. 690). DuMouchel (1973) estimated the asymptotic values and standard deviations of the stable distribution parameters, α, c, δ, and β. He pointed out the loss of information due to the use of censored samples. Koutrouvelis (1980, 1981) proposed a two-step regression method to estimate the parameters of a stable distribution. Following this procedure, α and c are estimated first. The second regression then uses the estimates of α and c to find δ and β. The advantage of this method is that it does not assume a symmetric distribution. McCulloch (1986) generalized the fractile method of Fama and Roll, and eliminated the asymptotic bias in the estimators of α and c. The approach of McCulloch(1986) is very direct, although it does assume a symmetric distribution. Janicki and Weron (1994) constructed approximations of stochastic integrals for their estimation of α-stable processes.

The stable distribution is of special interest in finance. Mandelbrot (1963a) pointed out that stock returns have tails fatter than the normal distribution. The occurrence of pronounced clusters or runs of consecutive large or small values observed in stock returns suggests persistence in the data which is inconsistent of the more typical assumption that stock returns are i.i.d. Further, the persistence in stock returns occurs across many different time scales which is indicative of long range memory (dependence). "The intensity of long-range dependence is related to the scaling exponent of the self-similar process." Doukhan, Oppenheim and Taqqu, (2003), p.5

Fama (1965) confirmed that monthly stock returns have a characteristic exponent of less than two. Teichmoeller (1971) concluded that the distribution of daily stock returns have fatter tails than that suggested by Fama. Using monthly data, Officer (1972) validated Fama's claim and pointed out the existence of the stability property in stock return distributions. Fieletz and Smith (1972) revealed that the stable distribution of stock returns is asymmetric. Based on the monthly Compustat data of the largest 200 industrials, Leitch and Paulson (1975) also demonstrated that stock returns have an asymmetric stable distribution. Applying his two-step regression method, Koutrouvelis (1980) agreed with Officer that the characteristic exponent of stock returns is close to 1.8. Using weekly data of stock returns to make a methodological comparison, Akgiray and Lamoureux (1989) chose Koutrouvelis' two-step regression method over the McCulloch's modified fractile method. They found that although McCulloch's modified fractile method and Koutrouvelis' two-step regression method provide similar estimates of α (alpha-characteristic exponent), the latter method had consistently smaller standard errors. Of 20 well known stocks examined the authors found the value of alpha to range from 1.5497 to 1.9303 with the regression method.

A few studies, however, have challenged the established view that stock returns follow a stable distribution. Blattberg and Gonedes (1974) believe that a Student-t distribution fits stock return data better. Hsu, Miller and Wichern (1974) and Hsu (1984) suggested that the existence of leptokurtosis in stock return distributions is attributable to samples with time-varying variances.

In addition to applications in stock returns, Cornew, Town and Crowson (1984) suggested that commodity futures have stable distributions. So (1987) used the McCulloch's modified fractile method to examine the price changes of foreign currency futures, and found that these price changes were well described by the L-stable distribution. The conclusion, that futures price changes have stable distributions, however, has been challenged by Hudson, Leuthold, and Sarassoro (1987), Hall, Brorsen, and Irwin (1989) and Gribbin, Harris and Lau (1992).

Jegadeesh (1990), Lehmann (1990), and Jegadeesh and Titman (1993) introduced the momentum effect. Since it cannot be properly explained by asset pricing theories, the sources of this effect have become a subject of academic debate. Persistence in stock returns has been offered by Conrad and Kaul (1998) as well as Chordia and Shivakumar (2002) as the cause of momentum effect. However, stock specific returns are suggested by Jegadeesh and Titman (1993) as well as Grundy and Martin (2001) as the root of the problem. Daniel, Hirshleifer, and Subrahmanyam (1998) examined the issue from the perspective of investment psychology. Moskowitz and Grinblatt (1999), on the other hand, believed that momentum effect varies across industry lines. According to Johnson (2002), the puzzling phenomenon is attributable to growth rates of individual firms, implying no irrationality in investment behavior.

Psychological research has demonstrated that investors usually rely on familiarity to reduce the complexity of financial analysis [Heath and Tversky (1991), Nofsinger (2002)]. Of two stocks with identical risk, an investor tends to select the one which is better known. As matter of fact, familiarity-bred stock selection cannot be explained by CAPM or any other risk based investment theory. Thus, investors in the United States tend to invest more in domestic stocks. There should be no surprise to learn that a significant portion of Coca-Cola's shares are owned by investors in Georgia, the home state of the company [Nofsinger (2002)]. As the size of a firm increases, investors become more familiar with its name. Many investors therefore are expected to actively trade shares of larger companies than small firms.

III. Methodology

Following Koutrouvelis (1980), (1981), a two-step regression method is used to estimate the parameters of a stable distribution. α and c are estimated first. These estimates are then used in the second regression to find δ and β. From Equation (1), we obtain

$$(2) \qquad \log(-\log|\varphi(t)|^2) = \log 2c^{\alpha} + \alpha \log|t|.$$

Let t assume the values of $t_k = \pi k/25$, k = 1, 2, …, K, where K is an appropriate integer optimally selected by Koutrouvelis (1980,1981) through simulation. Denote $y_k = \log(-\log|\varphi(t)|^2)$, $\mu = \log 2c^{\alpha}$, and $w_k = \log|t|$. Equation (2) can be approximated as

$$(3) \qquad y_k = \mu + \alpha w_k + \varepsilon_k, \qquad \text{for } k = 1, 2, \ldots, K,$$

where the errors ε_k are assumed to be iid with mean zero. This provides the first regression equation.

Using the original observations x_j ($j = 1, 2, \ldots, n$) to calculate $\varphi(t)$ and then y_k, and following the procedures proposed by Fama and Roll (1968, 1971), we obtain estimates of preliminary c_0 and δ_0 . After standardizing our data $x'_j = (x_j - \delta_0) / c_0$, ($j = 1, 2, \ldots, n$), apply the new data to estimate μ and α by regressing y_k on w_k, and then solve for c, as suggested by Equation (3). Denote the estimates of α and c by α_1 and c_1, respectively. The final estimates of these parameters are $\alpha^* = \alpha_1$ and $c^* = c_0\, c_1$.

Denote the real part of $\varphi(t)$ by $\mathrm{Re}\varphi(t)$, and the imaginary part by $\mathrm{Im}\varphi(t)$. These two parts of $\varphi(t)$ can be stated as follows:

(4) $$\mathrm{Re}\varphi(t) = \exp(-|ct|^{\alpha}) \cos[\delta t - |ct|^{\alpha}\beta \,\mathrm{sgn}(t) \tan(\pi\alpha/2)],$$

and

(5) $$\mathrm{Im}\varphi(t) = \exp(-|ct|^{\alpha}) \sin[\delta t - |ct|^{\alpha}\beta \,\mathrm{sgn}(t) \tan(\pi\alpha/2)].$$

From Equations (4) and (5), we obtain

(6) $$\arctan(\mathrm{Im}\varphi(t) / \mathrm{Re}\varphi(t)) = \delta t - \beta c^{\alpha} \tan(\pi\alpha/2)\, \mathrm{sgn}(t)\, |t|^{\alpha}.$$

For the purpose of estimating δ and β, let z_l denote the value of $\arctan(\mathrm{Im}\varphi(t) / \mathrm{Re}\varphi(t))$, and t assumes the values of $u_l = \pi l / 50$, for $l = 1, 2, \ldots, L$, where L is an appropriate integer optimally determined by sample size n and the estimated α. The data used for the first regression, x'_j, is transformed again to $s_j = (x'_j / c_1) - \delta_c$, for $j = 1, 2, \ldots, n$, where δ_c is a constant that makes u_l continuous in the interval [0, πl / 50]. Now, s_j's have a scale close to one, and Equation (6) can be restated as

$$(7) \qquad z_l = \delta u_l - \beta \tan(\pi\alpha/2)\, \mathrm{sgn}(u_l)\, | u_l |^{\alpha} + \eta_t,\ l = 1, 2, \ldots, L,$$

where the errors, η_t, are assumed to be iid with mean zero.

Equation (7) is used as the model for our second-step regression. Regressing z_l on u_l and $\tan(\pi\alpha/2)\, \mathrm{sgn}(u_l)\, | u_l |^{\alpha}$, we obtain δ_2 and β_2 as the estimates of δ and β. The final estimates of these parameters are $\delta^* = \delta_0 + c^*(\delta_2 + \delta_c)$ and $\beta^* = \beta_2$. The estimators, α^*, c^*, δ^*, and β^*, are consistent asymptotically unbiased as well as efficient.

IV. Data and Findings

As pointed out above, we use NYSE monthly return data from 1926-1997 obtained from CRSP. Our study considers index return data reported for all 10 deciles in the CRSP database.

Table 1 shows the CRSP monthly index returns by deciles, 1926-1997. Firms listed on the NYSE are ranked by their market values. Decile 1 covers the smallest 10% of these firms, while Decile 10 includes the largest 10% of these firms. The distribution of stock returns in each decile is obviously not normal. The histograms for all ten deciles as well as a random normal are included at the end of this paper. Out of the ten deciles, only Decile 2 has the mean return within 10% of the median return. The skewness for all deciles is four to seven times larger than what would be expected for a normal distribution (an example of which is shown in the Figure 11). In addition, kurtosis is three to eight times larger for all of deciles than what would be expected for a normal distribution, as shown in Figure11. The Jarque-Bera statistic rejects the hypothesis of normality at a 99.9% confidence level.

Table 1 also shows that the standard deviation decreases as firm size moves from Decile 1 towards Decile 10. Using standard deviation as a measure, smaller firms therefore have higher risk than larger firms. This change in risk across deciles is compensated with a change in return. Therefore, the expected return of Decile 1 is higher than the expected return of Decile 10.

Table 2 presents additional indications of a non-Gaussian distribution. We report the values of the 1% truncated mean, 25% truncated mean, 50% truncated mean, 75% truncated mean and the 99% truncated mean. If stock returns were normally distributed, the expected values of all of these truncated means would be equal to the sample mean.

In Table 3, the characteristic exponent (α) for each decile of stock returns is estimated. Note that the correspondence between the fatter-tails in Figure 1 with the smaller values of α. The t-statistics and adjusted - R^2 values indicate a high degree of confidence in the estimation of α. Given 864 data points for each of our deciles, we would expect a very good representation of the underlying sample distribution. We note that α increases monotonically with each of the deciles considered. This is also the case for both the measure of skewness and Kurtosis.

Long memory or long-range dependence is often measure by the self-similarity index H, which is equal to $1/\alpha$. When $½ < H < 1$, a process exhibits long-range dependence. That is, a time series has an autocovarance function which decreases slowly [Samorodnitsky and Taqqu (1994)]. As α approaches 2, H becomes closer to ½, and the process approaches a normal distribution.

When stock returns have a non-normal distribution, their standard deviation is likely to reflect the long memory in returns. As the long memory of stock returns decreases with firm size, the standard deviation changes across deciles. Non-normal stable distributions are characterized with fat tails, skewness and leptokurticity. The smaller is the characteristic exponent, the thicker the tails, and the higher the leptokurticity.

According to Samuelson's fundamental approximation theorem, if stock returns are normally distributed, risk, measured by standard deviation or variance, is often considered as the primary determinant of expected return. When the distribution of returns is not normal, expected returns should also reflect the impacts of higher moments. For this reason, as suggested by Booth and Smith (1987), the size effect should take skewness and leptokurtosis into consideration. Since traditional risk measures do not take higher moments into consideration, the momentum effect may not be adequately explained by asset pricing models unless the risk premium can reflect the impacts of higher moments.

Although the momentum effect may be attributable to long memory of stock returns, it begs the question as to why the long range dependence varies across deciles. The answer may lie in investment psychology and behavior. Unlike the omniscient economic agent described by mainstream investment theory, investors often rely on shortcuts to process complicated, and sometimes indigestible, financial information. When an investor does not have the time and/or capability to process and to comprehend the financials of thousand of companies, he/she may use a psychological tool – familiarity [Heath and Tversky (1991), Nofsinger (2002)]. Stocks of larger companies are better known, and they are more actively traded because many investors make their decisions on familiarity, not measures of risk or moments. Frequent trading reduces the possibility of continuation of short-term returns. Consequently, returns on large-firm stocks may be less likely to exhibit as much long term persistence as small-firm stocks.

The impact of familiarity-bred investment decisions is magnified when herding is observed in the market. Instead of making a thorough analysis of financial data, an investor often follows the recommendation of an advisor or the decision of a group. The participation of an investment club can be viewed as an example of this investment psychology. Market mania is at least partially fueled by herding. Thus, the differences in return persistence between deciles due to familiarity are further magnified by the existence of socialization of investment behavior.

VI. Conclusion

Over the period 1926-1997, we have found long-term persistence in stock returns. We have also found that the values of α increase monotonically with the size of the firm considered. The returns of large size firms do not depart from the Gaussian distribution to the same degree as small size firms. However, even in the case of large size firms, the degree of skewness, leptokurtosis, and the value of α (characteristic exponent) lead us to the conclusion that stock returns are better represented by the L-stable distribution than they are by the normal. Long memory of stock returns, as measured by $H = 1/\alpha$, is a source of momentum effect. Since α varies across deciles, the extent of momentum effect changes with firm size.

Standard deviation constitutes the basis for compiling risk measures. When stock returns have a non-normal stable distribution, their higher moments must be included for determining the risk premium. Since second and higher moments of a non-normal stable distribution are not well defined, any use of traditional risk measures to test an asset pricing theory is less than adequate. Thus, theoretically, momentum effect cannot be properly explained by an asset pricing model.

What then is the cause of firm-size related long range dependence in stock returns? A plausible explanation is investment psychology. Investors often make decisions based on their familiarity with individual companies. Familiarity with larger firms results in their frequent trading, and therefore the shortening of their stock return memory. That is, long range persistence in returns tends to decrease with firm size. In addition, the impact of familiarity-based individual behavior is magnified by herding – an investing psychology of investors as a group. If so, we may have gained some additional insight into the momentum-effect enigma.

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Table 1

NYSE Monthly Stock Returns, January 1926 – December 1997

Sample: 1926:01 1997:12

	Decile1	Decile2	Decile3	Decile4	Decile5	Decile6
Mean	0.016919	0.013922	0.012468	0.012334	0.011988	0.012048
Median	0.010040	0.013303	0.014022	0.014628	0.014296	0.015240
Maximum	1.059379	0.873956	0.893980	0.680514	0.583368	0.528056
Minimum	-0.358667	-0.350549	-0.341763	-0.314534	-0.328280	-0.323182
Std. Dev.	0.107410	0.091926	0.082645	0.076121	0.071841	0.070029
Skewness	3.064073	2.274792	2.146286	1.283569	1.041551	0.875396
Kurtosis	27.10170	23.80836	25.85406	16.76276	15.43878	13.81819
Jarque-Bera	22264.06	16332.71	19466.44	7056.132	5726.257	4323.550
Probability	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
Observations	864	864	864	864	864	864

	Decile7	Decile8	Decile9	Decile10	Random/12
Mean	0.011602	0.010597	0.010711	0.009421	-0.000217
Median	0.014523	0.014070	0.014547	0.011912	-0.002994
Maximum	0.574526	0.495114	0.459032	0.351162	0.241453
Minimum	-0.307210	-0.316103	-0.324906	-0.278341	-0.261790
Std. Dev.	0.066137	0.062332	0.059869	0.051887	0.080285
Skewness	0.810250	0.514250	0.566568	0.165765	0.045603
Kurtosis	14.55632	12.65839	13.04560	10.60397	3.102645
Jarque-Bera	4902.288	3396.322	3679.128	2085.491	0.678768
Probability	0.000000	0.000000	0.000000	0.000000	0.712209
Observations	864	864	864	864	864

Table 2

NYSE Monthly Stock Returns, January 1926 - December 1997

	Trunc 1	Trunc 25	Trunc 50	Trunc 75	Trunc 99	Mean	Median	Std. Dev.
Decile 1	0.0098184	0.0104883	0.010755	0.0109556	0.0138583	0.0169186	0.010040	0.1074102
Decile 2	0.0132654	0.0126087	0.0122508	0.0118614	0.0117464	0.0139223	0.013303	0.0919264
Decile 3	0.0139874	0.0137369	0.0126625	0.0121204	0.0106806	0.0124685	0.014022	0.0826452
Decile 4	0.0146012	0.0146829	0.013784	0.0129946	0.0109815	0.0123343	0.014628	0.0761213
Decile 5	0.0143036	0.0142103	0.0136161	0.0128628	0.0107998	0.0119885	0.014296	0.0718412
Decile 6	0.0144400	0.0143187	0.0133461	0.0126263	0.0106977	0.0116019	0.015240	0.0700288
Decile 7	0.0144400	0.0143187	0.0133461	0.0126263	0.0106977	0.0116019	0.014523	0.0661367
Decile 8	0.0140977	0.0135777	0.0126993	0.0118869	0.0098749	0.0105972	0.014070	0.0623316
Decile 9	0.0145998	0.0138894	0.0127232	0.0119615	0.0099614	0.0107110	0.014547	0.0598695
Decile 10	0.011853	0.0117386	0.0114209	0.0108047	0.0089537	0.0094205	0.011912	0.0518869

Table 3

Characteristic Exponents (α) NYSE, January 1926 - December 1997

	Alpha α	t - statistics	Adj - R^2
Decile 1	1.454706	72.681	0.9981
Decile 2	1.467179	53.053	0.9965
Decile 3	1.569274	106.276	0.9991
Decile 4	1.588701	91.434	0.9988
Decile 5	1.609601	127.177	0.9994
Decile 6	1.640964	148.950	0.9995
Decile 7	1.650368	131.606	0.9994
Decile 8	1.678612	126.230	0.9994
Decile 9	1.686580	122.658	0.9993
Decile 10	1.711025	107.248	0.9991

Figure 1 (Decile 1: NYSE Monthly Stock Returns 1/26 – 12/97)

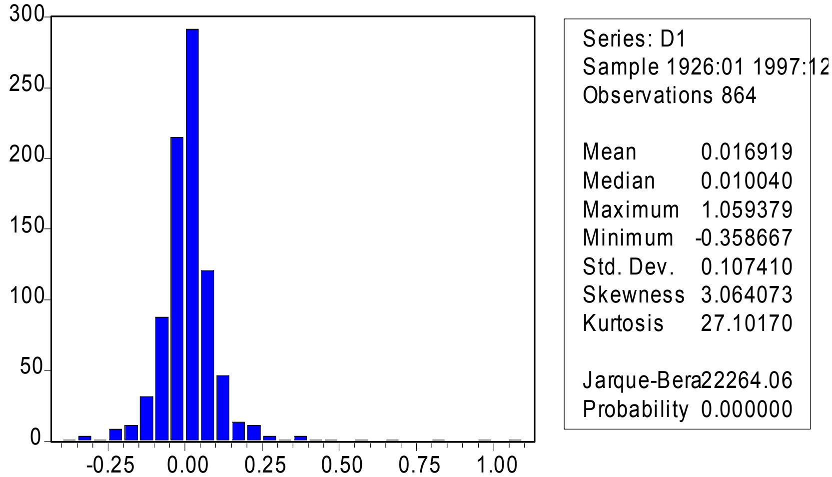

Figure 2 (Decile 2: NYSE Monthly Stock Returns 1/26 – 12/97)

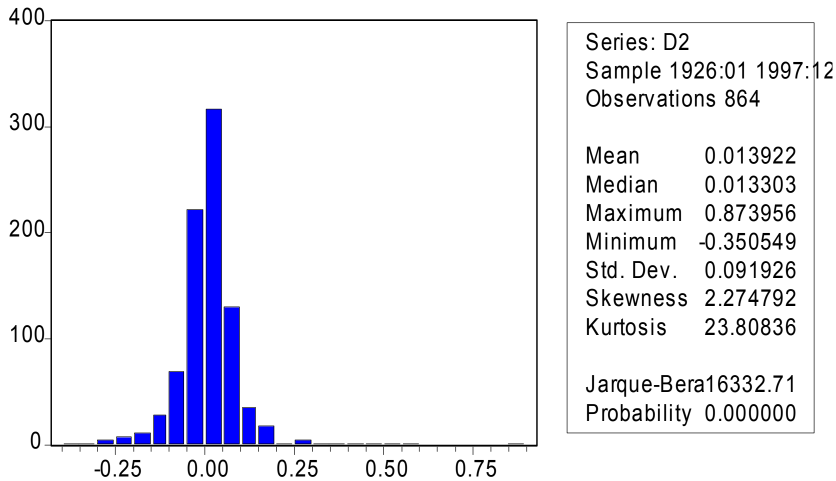

Figure 3 (Decile 3: NYSE Monthly Stock Returns 1/26 – 12/97)

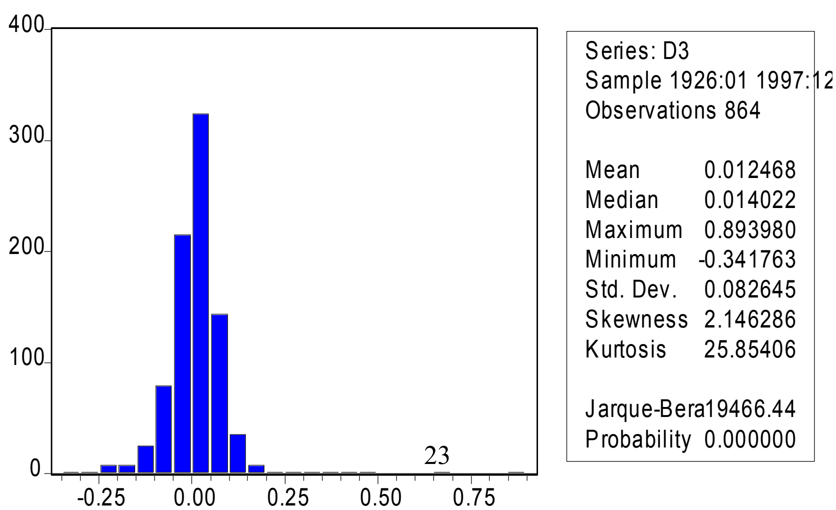

Figure 4 (Decile 4: NYSE Monthly Stock Returns 1/26 – 12/97)

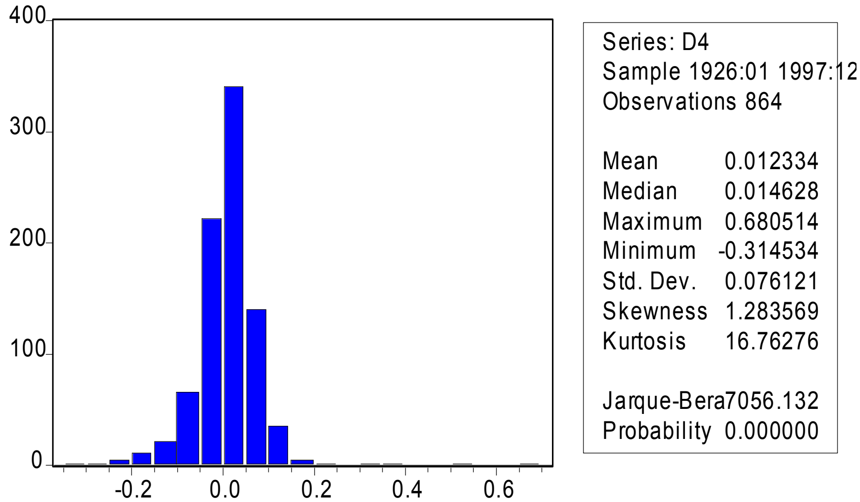

Figure 5 (Decile 5: NYSE Monthly Stock Returns 1/26 – 12/97)

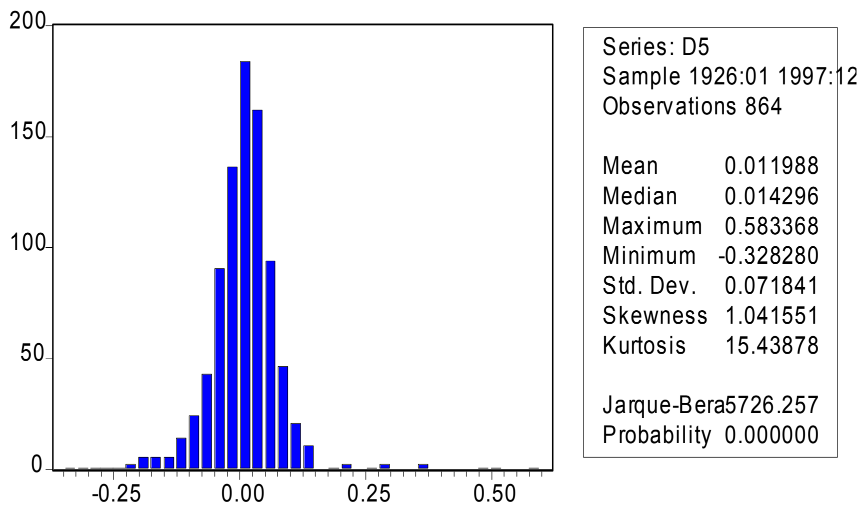

Figure 6 (Decile 6: NYSE Monthly Stock Returns 1/26 – 12/97)

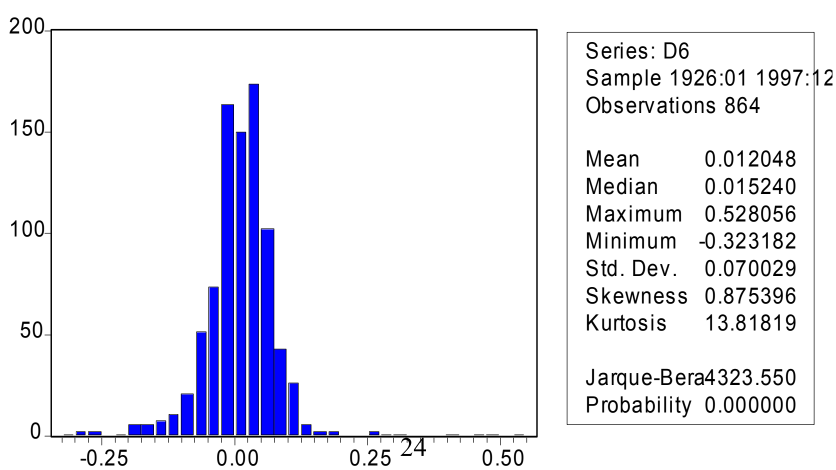

Figure 7 (Decile 7): NYSE Monthly Stock Returns 1/26 – 12/97)

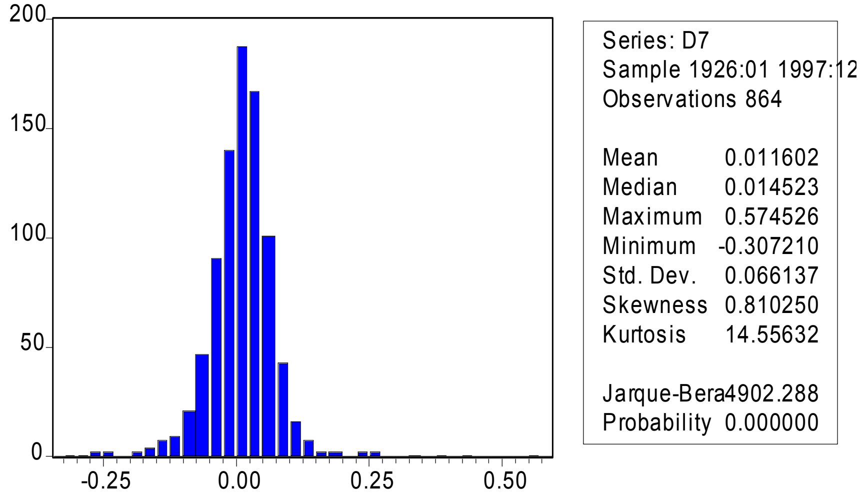

Figure 8 (Decile 8): NYSE Monthly Stock Returns 1/26 – 12/97)

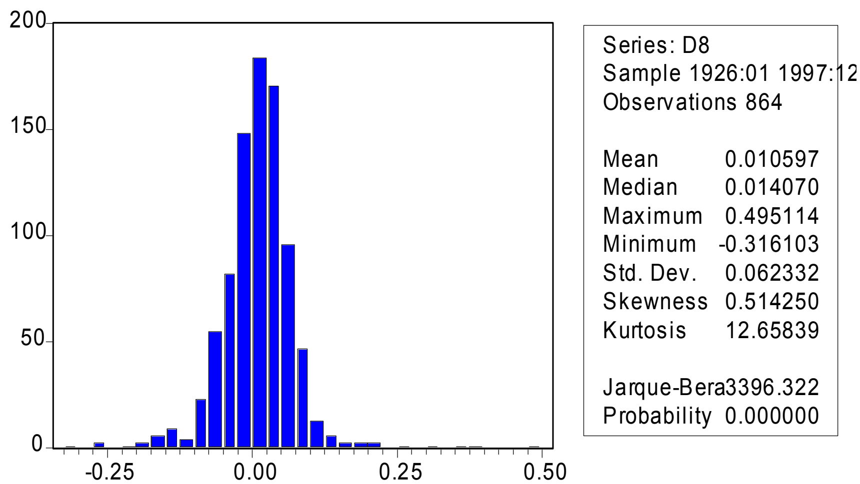

Figure 9 (Decile 9): NYSE Monthly Stock Returns 1/26 – 12/97)

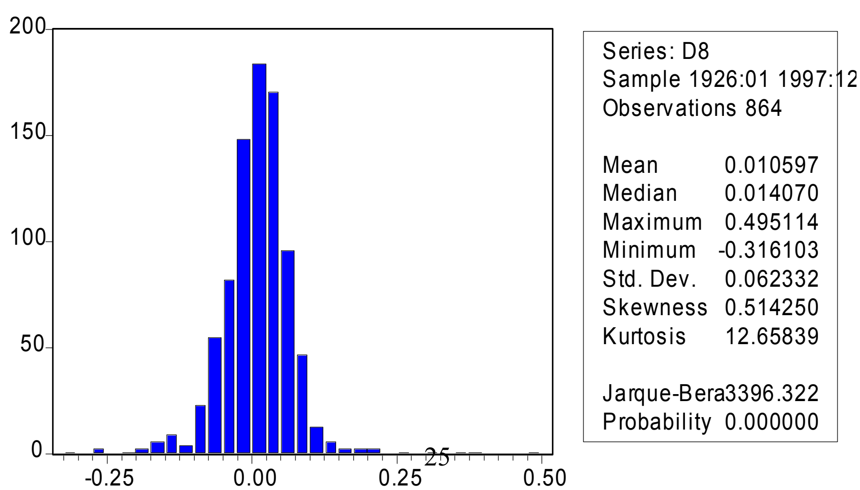

Figure 10 (Decile 10: NYSE Monthly Stock Returns 1/26 – 12/97)

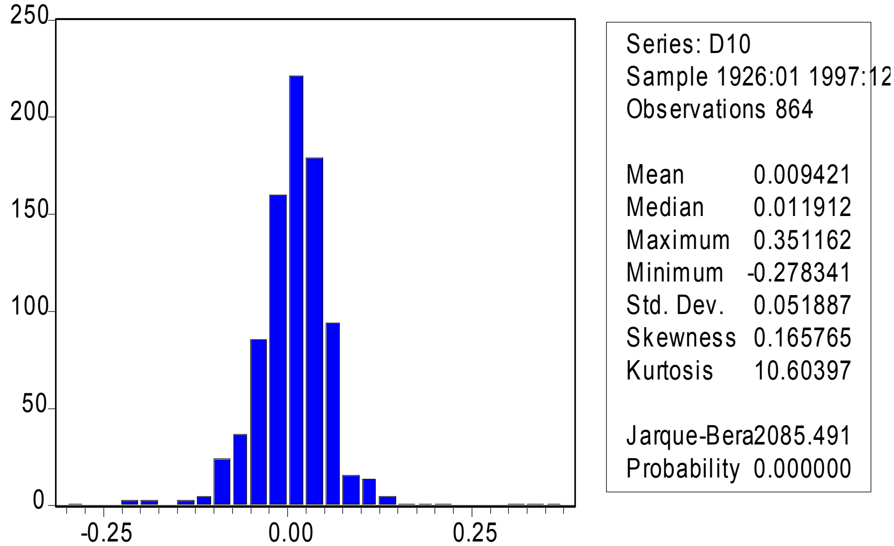

Figure 11 (Rand/12: Random-Normal 0, 1 / 12)