Multifractal nature of stock exchange prices

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Abstract

The multifractal structure of the temporal dependence of the Deutsche Aktienindex (DAX) is analyzed. The q-th order moments of the structure functions and the singular measures are calculated. The generalized Hurst exponent $H(q)$ and the $h(\gamma(q))$ curve indicate a hierarchy of power law exponents. This approach leads to characterizing the nonstationarity and intermittency pertinent to such financial signals, indicating differences with turbulence data. A list of results on turbulence and financial markets is presented for asserting the analogy.

Key words: time series analysis, fractals
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1 Introduction

The analogy between fluid turbulence and financial markets has been previously noticed [1,2]. The energy flow in hydrodynamics is supposed to mimic the information transfer in financial markets, or the entropy variation of the market [3]. Some hierarchical structures are thought to exist, leading to cascades of information and clusters of buy-sell orders [4,5] and sometimes to crashes [6]. Surprisingly, more work on "financial cascades" can be found on the foreign exchange market time series [7-14] rather than on share prices or stock market indices. However, since all such financial time series indicate nonstationarity, a multifractal [15,16] description seems useful [13,14,17-24]. The goal of this paper is to present original results on the Deutsche Aktienindex (DAX), obtaining the roughness parameter (H_1) and the degree of intermittency (C_1). The q-th order moments of the structure functions and the singular measures are constructed thereof [23-25]. The behaviors are consistent with the multi-affine properties of other turbulent phenomena [26]. Understanding the processes that underlie these "macroscopic" effects remains at the speculative level. Numerical differences are pointed out. Nevertheless, the nonexhaustive list of references is a sufficient argument for asserting the analogy and suggesting further work.

2 Experimental data analysis

The Deutsche Aktienindex (DAX) data used here is from $http://deutsche-boerse.com/$. It goes from Oct. 01, 1959 till Dec. 30, 1998. Let the data consist in a series $y(t)$, where t is a discrete variable t_i, i.e. $N = 9818$ data points. First, the scaling range (if any) is found from a detrended fluctuation analysis (DFA) method [27]. It is found that the scaling range extends up to 256 days, after which the error bars become too large. The scaling (Hausdorff) exponent $Ha \sim 0.54$ is about the same as that of the DJIA [9]. It is expected that the self-affine fractal dimension $D = 2 - Ha$. Knowing the scaling range, one can

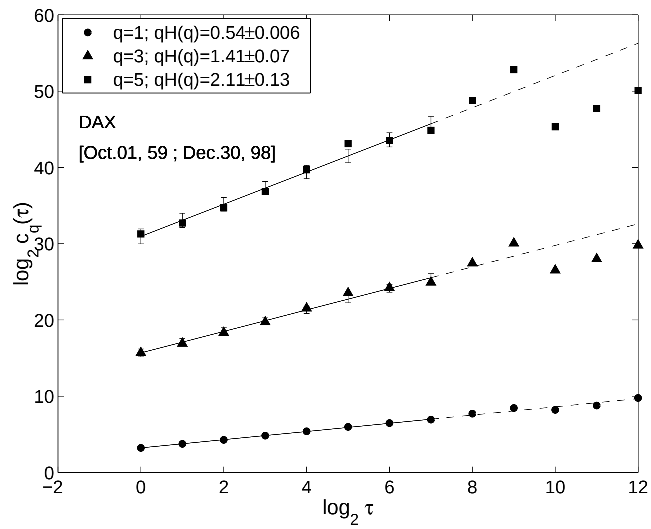

Fig. 1. The DAX q-th order structure functions $c_q(\tau)$ as a function of time lag τ during Oct. 01, 59 - Dec. 30, 98

reconsider the problem of the time variation of D (or Ha). In the multifractal approach [9,23,26], one seeks for various scales of self-affinity, through the so-called q-th order structure functions

$$c_q(\tau) = \langle |y(t_{i+r}) - y(t_i)|^q \rangle_\tau \qquad i = 1, 2, \ldots, N - r \tag{1}$$

where only non-zero terms are considered in the average $\langle ... \rangle_\tau$ taken over all $N - r$ couples (t, t') such that $\tau = |t - t'|$ is a characteristic time lag, $\tau = t_{i+r} - t_i$, with $r \geq 0$, see Fig.1. Assuming a power law dependence of the structure function $c_q(\tau)$, the $H(q)$ spectrum is defined through

$$c_q(\tau) \sim \tau^{qH(q)} \qquad q \geq 0, \tag{2}$$

and is shown in Fig.2, as $qH(q)$; note that $Ha = H(1)$ [22]. The *intermittency* of the signal can be studied through a singular measure analysis. Define a measure $\varepsilon(1;i)$ as

$$\varepsilon(1;i) = \frac{|\Delta y(1;i)|}{< \Delta y(1;i) >}, \qquad i = 0, 1, \dots, N-1 \tag{3}$$

where $\Delta y(1;i) = y(t_{i+1}) - y(t_i)$ is the small-scale gradient field and

$$< \Delta y(1;i) >= \frac{1}{N} \sum_{i=0}^{N-1} |\Delta y(1;i)|. \tag{4}$$

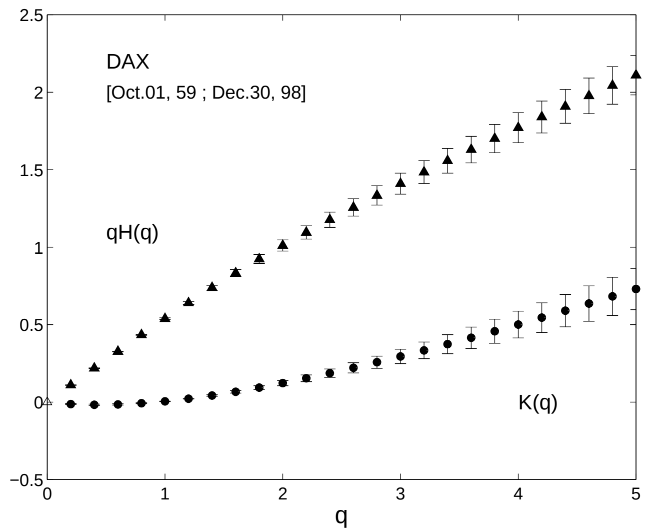

Fig. 2. The DAX $qH(q)$ and $K(q)$ scaling functions for [Oct. 01, 59-Dec. 30, 98]

Next, define a series of ever more coarse-grained and ever shorter fields $\varepsilon(r;l)$ where $0 < l < N - r$ and $r = 1, 2, 4, \dots, N = 2^m$. The average measure in the interval $[l; l + r]$ is

$$\varepsilon(r;l) = \frac{1}{r} \sum_{l'=l}^{l+r-1} \varepsilon(1;l') \qquad l = 0, \dots, N - r \tag{5}$$

The scaling properties are then searched for through $\chi_q(\tau) =< \varepsilon(r;l)^q >_\tau \sim \tau^{-K(q)}$ for $q \geq 0$. Thereby, the multifractal properties of the DAX signal are expressed by two scaling functions, $H(q)$ for describing the roughness of the signal and $K(q)$ its intermittency. The $K(q)$ spectrum (Fig.2) is closely related to the generalized dimensions $D_q = 1 - K(q)/(q-1)$ [15,16]. A nonlinearity of both $qH(q)$ and $K(q)$ implies multifractality. Let $C_1 = -\left|dK(q)/dq\right|_{q=1}$. It seems to be a measure of the information entropy of the system [3]. For the DAX, $C_1 = 0.07 \pm 0.002$, interestingly compared to $C_1 = 0.27$ for the DJIA [9].

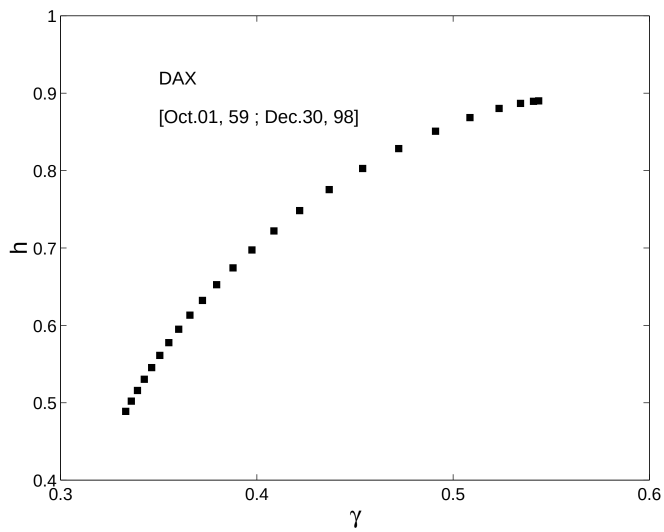

Fig. 3. The $h(\gamma)$ curve for the DAX data from Oct. 01, 1959 to Dec. 30 1998

Note that both Ha's $= 0.54 \pm 0.006$ are similar. Let $\gamma(q) = d(qH(q))/dq$ and $h(\gamma(q)) = 1 + q\gamma(q) - qH(q)$. The function $h(\gamma(q))$ is the fractal dimension of the Cantor dust set of points having the same roughness exponent $\gamma(q)$ [16]. The $h(\gamma)$ function acts like the function $f(\alpha)$ in Ref.[16]. The $h(\gamma(q))$ curve is shown in Fig. 3. The error bars are easily estimated from Fig.1 and the above equations. Note that $h(\gamma(q))$ reaches a maximal dimension for some finite γ_0 corresponding to the fractal dimension of the signal if it was self-affine [22].

3 Discussion

For a long time, the only available theoretical background to the statistical behavior of prices was the Efficient Market Theory (EMT). The EMT has been usually identified with the random walk character of prices ($Ha = 0.5$, $C_1 = 0$). In finance papers, $Ha > 0.5$ values have been reported , -the deviations from 0.5 being explained by regulations imposed on the market by central authorities. However, Ha seems to be often different from 0.5, and C_1 is finite. Thus, one should expect multifractal features indicating that the origin of scaling laws in financial time series is to be found in exogenous forces that cover a wide variety of influences. Finally, let us conclude that fluctuations in DAX and other financial market data time series [8,9] can be compared to those occurring in turbulence. Analogies with intermittency, cascades, period doubling [21] can be invoked with Kolmogorov 1/3-law process [28] and the fractional Brownian motion as basic ideas. Recall that intermittency as introduced in turbulence led to a multifractal description, with $Ha = 1/3$ and $C_1 \sim 0.05$.The above data indicate that the DAX (and DJIA [9]) $H_1 \sim 0.5$ and $C_1 \sim 0.07$ are far away from the turbulence domain $H_1 \sim 0.3$ and $C_1 \sim 0.05$.

Thus there is some analogy, but models should be different!

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Note added in proofs :

A surrogate data analysis has been performed as well. The corresponding values of $qH(q)$ as found in Fig.1 are 0.05 ± 0.005; 0.12 ± 0.03; 0.19 ± 0.06, respectively.

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