Why Derivatives on Derivatives?
The Case of Spread Futures

Charles J. Cuny
Texas A&M University
College Station, TX 77843-4218
(979) 845-3656
ccuny@cgsb.tamu.edu

November 2003

I thank the Chicago Board of Trade (Market Data Products and Information Department) for providing data.

Why Derivatives on Derivatives? The Case of Spread Futures

Abstract

Recently, calendar spread futures, futures contracts whose underlying asset is the difference of two futures contracts with different delivery dates, have been successfully introduced for a number of financial futures contracts traded on the Chicago Board of Trade. A spread futures contract is not an obvious financial innovation, as it is a derivative on a derivative security: a spread futures position can be replicated by taking positions in the two underlying futures contracts, both of which may already be quite liquid. This paper provides a motivation for this innovation, demonstrating how the introduction of spread futures can, by changing the relative trading patterns of hedgers and informed traders, affect equilibrium bid-ask spreads, improve hedger welfare, and potentially improve market-maker expected profits.

In January 2001, Alliance/CBOT/Eurex (a/c/e), a joint venture of the Chicago Board of Trade and Eurex futures exchanges, began trading four separate reduced tick spread futures contracts. These futures contracts are traded exclusively on the a/c/e electronic trading platform, while their clearing is through the CBOT system. The underlying asset for these futures is a calendar spread position across two otherwise identical CBOT futures contracts with adjacent delivery dates. Thus, these spread futures contracts are redundant securities in the sense that the contract essentially consists of a long position in one futures contract, and a short position in another futures contract, with the two legs of the spread differing only in their delivery dates.

For example, the March 10-year US Treasury Note Futures Reduced Tick Spread futures contract has a trading unit that consists of "one March-June Ten-year US Treasury Note futures spread having a face value at maturity of $100,000 or multiple thereof"[1]. Taking a long position in this spread futures contract immediately gives the trader a long position in the CBOT June 10-year US Treasury Note futures, and a short position in the CBOT March 10-year US Treasury Note futures, just as if these two positions were entered into directly, but separately, through the CBOT Treasury Note futures markets. Thus, for clearing purposes, a trade executed in a spread futures contract is recognized as being exactly the same as two trades in the futures contracts corresponding to the legs of the calendar spread.

In another, narrower sense, reduced tick spread futures contracts are perhaps not totally redundant: although the tick size of the CBOT Mar 10-Year US Treasury Note futures contract is one-half of 1/32 of a point, the tick size of the associated a/c/e Reduced Tick Spread futures contract is, as implied by its name, smaller: one-quarter of 1/32 of a point.[2] Thus, reduced tick spread futures can trade at prices unavailable by taking the associated direct long and short positions in the underlying futures markets.[3]

[1] Language taken from the description on the Chicago Board of Trade website www.cbot.com
[2] The tick size for the CBOT Treasury note (both 10-year and 5-year) and Agency note (both 10-year and 5-year) futures contracts is one-half of 1/32 of one point. The tick size for the CBOT Treasury bond and Interest rate swap (both 10-year and 5-year) futures contracts is 1/32 of one point. The tick size of all the associated reduced tick spread futures is one-quarter of 1/32 of one point. Thus, the tick size for each spread futures contract is one-half or one-quarter the tick size of the associated futures contracts.
[3] Of course, if reducing the tick size is the primary innovation associated with these spread futures, one could ask why not instead reduce the tick size on the underlying CBOT futures contracts.

Because the spread futures contract price reflects the price differential between two futures contracts (differing in delivery date), one would naturally expect the spread futures price to be much smaller than the price of either leg of the underlying spread. In light of this, implementing the reduced tick pricing may seem quite natural. Furthermore, the spread differential may even carry a negative price, depending upon the slope of the interest-rate term structure, for the case of interest-rate futures.[4]

However, since spread futures contracts are redundant, in the sense that the underlying spread could be straightforwardly traded on other futures markets, it is not immediately clear why this particular financial innovation should be successful. Indeed, not only are spread futures contracts a derivative of another derivative (that is, the two underlying futures contracts composing the calendar spread), but the particular spread futures introduced to date are based on very liquid futures contracts. This paper examines how the introduction of this apparently redundant security can nevertheless change the trading behavior and welfare of hedgers in such a way as to make this innovation potentially attractive to a futures exchange.

As of now, a/c/e has introduced seven reduced tick spread futures contracts. Three of these, with underlying of US Treasury Bond futures, 10-year Treasury Note futures, and 5-year Treasury Note futures, were introduced in January 2001, and have been well received in the marketplace. A 10-year Agency Note futures reduced tick spread futures contract was also introduced in January 2001. Although modestly successful at first,[5] the trading volume for both the underlying 10-year Agency Note futures and its associated reduced tick spread futures have migrated to the 10-year Swap futures and its associated reduced tick spread futures since the CBOT introduced the 10-year Swap futures contract in October 2001 (followed by its reduced tick spread futures contract in May 2002). Two other reduced tick spread futures products, based on the 5-year Agency Note futures and 5-year Swap futures contracts, have also been introduced. However, neither of the two underlying futures contracts nor their associated reduced tick spread futures contracts

[4] a/c/e has developed a pricing convention responding to traders' presumed disinclination to work with negative prices. The convention is based on adding 100 basis points to the price differential for all reduced tick spread futures prices.

[5] The underlying 10-year Agency Note futures contract has also achieved only modest success since its introduction in 2000.

have ever generated meaningful volume up to this time. Figure 1 shows the monthly trading volume to date for the first five reduced tick spread futures contracts.

As is apparent from Figure 1, there is seasonality in the volume for each of these contracts. The expiration month on all the underlying financial futures contracts is March, June, September, or December. The significant demand for rolling over futures contracts occurs in four weeks leading up to the contract expiration date. Closer examination of daily trading volume reveals that, for each of these reduced tick spread futures, volume starts to visibly increase around the 15th to 20th of the previous month, typically peaks on the 28th or 29th, then visibly decreases around the 7th of the contract month. (For example, for the March contract, the largest volume occurs from about February 15 through March 7.)

This paper shows how the structure of the transaction costs (modeled in the form of bid-ask spreads) in the futures market is changed by the introduction of calendar spread futures. The futures markets exist in the model to service hedging demand, an approach traceable to Working (1953). Also present are informed traders. Market-makers provide liquidity to the market, which is costly to provide due to the adverse selection of facing informed traders. Market-makers are compensated for this through charging traders a bid-ask spread. With competitive market-makers, bid-ask spreads in each contract are set to just cover the adverse selection faced by market-makers in that contract. However, if the cost of trading a calendar spread is lower in the spread futures than the primary futures market, then hedgers' trades will partially migrate to the spread futures market, leaving informed trading in the primary market. Furthermore, if the overall cost of implementing hedging trades falls, additional hedging interest may arise in the primary market.

Introducing spread futures allows partial separation of hedging and informed trading. Trading in the spread futures market is concentrated in hedging, and therefore supports a lower bid-ask spread. To implement this requires offering a finer pricing structure, or a reduced tick size, in the spread futures relative to the primary futures market.

A similar result may be obtained if bid-ask spreads are set by an exchange exercising pricing power in order to maximize aggregate market-maker profit. With these "monopolistic" bid-ask spreads, it is optimal to lower the bid-ask spread in the calendar spread futures, attracting hedgers, while raising the bid-ask spread in the primary market. This allows price discrimination between hedgers and informed traders. Informed traders face higher costs of trading, and reduce their activity, moderating the adverse selection problem faced by the market-makers. The lower bid-ask spread in the spread futures market effectively subsidizes hedgers, keeping their overall trading cost relatively low in order to generate the hedging trade activity that market-makers find profitable. Since hedgers have the opportunity to adjust their trading to respond to the new trading cost structure, there will be a movement toward markets featuring relatively lower trading costs. In particular, since spreading is now more cost effective, hedgers are more willing to enter into initial positions that may require a later portfolio adjustment utilizing spread futures. This trading adjustment allows for a decrease in the aggregate cost of trading borne by the hedgers.

An important factor in the implementation of either such equilibrium, offering a lower bid-ask spread for direct trading of the calendar spread, is that the spread futures contract trades with a smaller tick size than the primary futures market. Even without the presence of the spread futures, a trader could, in principle, negotiate both legs of the calendar spread simultaneously in the futures market. However, the possible prices at which the legs can be negotiated is constrained by the tick size. By allowing a smaller tick size, the presence of the spread futures market allows a finer set of possible prices, and therefore a smaller bid-ask spread (lower transaction cost) in trading the calendar spread.

Excellent overviews of the literature on financial innovation are provided in Allen and Gale (1994) and Duffie and Rahi (1995). Specific cases of innovations of futures contracts are discussed in Working (1953), Gray (1970), Sandor (1973), Silber (1981), and Johnston and McConnell (1989).

This paper reaches the conclusion that the introduction of spread futures, which appear to be a redundant security, can change trading patterns and hedger welfare. In the options literature, there is evidence that options may not be truly redundant. Conrad (1989) finds a significantly positive (two percent) abnormal stock return accompanying the introduction of stock option trading for listings from 1974 and 1980. Detemple and

Jorion (1990) find similar abnormal stock returns for listings from 1973 to 1982, but no significant effect for listings from 1982 to 1986. Back (1993) models a market where an option can be synthesized via dynamic trading, thus appearing to be redundant, but the option's existence affects the information flow, making the underlying asset volatility stochastic. Longstaff (1995), for the case of S&P 100 index options, rejects the martingale restriction that the value of the underlying asset implied by the cross-section of options prices equals its actual market value, finding that the differences in value is related to market frictions. If markets are dynamically complete and options are redundant assets, then any option payoff can be replicated using the underlying asset and one additional option. Buraschi and Jackwerth (2001) perform this direct test, concluding that at-the-money S&P 500 index options and the underlying index do not span the pricing space; consequently, the options are not redundant securities. Bakshi, Cao, and Chen (2000) conclude that index options are not redundant assets, as the index level and associated call option prices often move in opposite directions.

There is also a literature on how the introduction of futures contracts may affect the underlying asset markets. A comprehensive summary of this literature is in Mayhew (2000). One related paper is Subrahmanyam (1991), which provides an information-based model for stock-index futures ("basket trading"), extending Kyle's (1984) model to allow simultaneous trading of individual stocks and baskets of stocks. With the introduction of trading the basket, uninformed traders tend to trade the basket to protect themselves from the informed traders who tend to trade individual stocks. (The informed traders have stock-specific information.) Thus, the model predicts liquidity will migrate from the markets for individual stocks to the basket.

1. The model

The model contains hedgers, informed traders, and market-makers in an overlapping generations-style marketplace. Market-makers provide liquidity to the futures markets by taking the opposite side of trades, as needed, and are compensated by capturing the difference between the bid and ask prices. The cost faced by market-makers is the adverse selection cost incurred when trading against better informed traders. Market-makers are risk neutral.

Time is broken up into a series of trading periods (dates). There is a risky asset, whose underlying value changes between each period, with mean zero and variance σ^2 (value changes are independent over time). A series of futures contracts are available, each of which is tradable for two consecutive periods, after which delivery takes place (although in our model, traders clear their positions before delivery occurs). Futures contracts overlap, so that in each trading period, two futures contracts with different delivery dates are extant: traders can take positions in the "new" contract (delivery immediately after next period), and close out positions in the "old" contract (delivery immediately after this period).

Each period, there is a mass H of new hedgers. Each hedger is equally likely to be endowed with +E or –E units of the risky asset. Each hedger's lifetime is either one period long (a fraction $1 - q$ of the hedgers) or two periods long (fraction q). A hedger can trade in the futures market(s) for the risky asset during his life. Thus, a hedger with a one period lifetime born at date T can initiate a futures position at date T, and close the position at date $T + 1$, while a hedger with a two period lifetime can initiate a futures position at date T, adjust it at date $T + 1$, and close it at date $T + 2$. Hedgers do not know their lifetime at birth, but find it out after one period passes. Hedgers consume during their final period of life, and have mean-variance utility with risk-aversion Γ over final consumption.

Each period, there is a mass I of new informed traders. Each informed trader has private information about the next periodic change in the value of the risky asset. Conditional on her information, an informed trader either appraises the next risky asset value change as having mean $+\theta > 0$ or mean $-\theta$ (equally likely, *ex ante*), with variance $k\sigma^2$. Therefore, an informed trader's gain from making a unit trade (of appropriate direction) in the risky asset has mean θ and variance $k\sigma^2$ (less bid-ask spread).[6] The parameters θ and k thus measure the mean and variance of the quality of informed

[6] One possible underlying structure is that the underlying risky asset value change is equally likely to be $+\sigma$ or $-\sigma$ each period, and the informed trader, after observing her signal, assigns probability $p > 1/2$ to the correct direction of price movement. Then $\theta = p\sigma - (1 - p)\sigma = (2p - 1)\sigma$, and $k = 4p(1 - p)$. Another possible underlying structure is that the asset value change each period takes the form $\Theta + \Phi$, with Θ equally likely to be $+\theta$ or $-\theta$, and Φ independent of Θ. If the informed perfectly observes Θ, then $k = 1 - (\theta/\sigma)^2$.

traders' information. Informed traders have one period lifetimes, consume during their final period of life, and have mean-variance utility with risk-aversion Γ over final consumption.

Two scenarios are considered: a scenario with the two already described futures contracts, new and old, trading each period ("without spread futures"), and an alternative scenario with a spread futures contract available as well ("with spread futures"). The available spread futures contract is based on the calendar spread between the new and old contracts, and is identical to a long position in the new contract combined with a short position in the old contract. The spread futures contract, when available to trade, will be the natural instrument for hedgers who discover their lifespan is two periods, leading to a desire to roll over their short-lived position into a longer-lived position. Utilizing spread futures allows these hedgers to convert their position in the old contract (which delivers before the conclusion of their hedging needs) into a position in the new contract (which delivers after the conclusion of their hedging needs). Of course, if the spread futures contract is unavailable, the hedgers can achieve a similar result directly by simultaneously trading equal and opposite positions in the new and old futures contracts.

Market-makers receive (and traders incur) a bid-ask spread in trading. The bid-ask spread is the frictional cost associated with a round trip trade for a hedger or informed trader. The endogenously determined bid-ask spread in the futures for the risky asset is denoted by S. When the spread futures are available, the bid-ask spread in the spread futures is denoted by D. Since the calendar spread can always be directly created in the futures markets, $D \leq S$.

Suppose that spread futures are available to trade. A hedger with endowment $-E$ optimally takes an initial futures position $x(S, D)$ satisfying[7]

$$\max_{x} \quad (1-q)[-S|x| - \Gamma\sigma^2(x-E)^2/2] + q[-(S+D)|x| - 2\Gamma\sigma^2(x-E)^2/2]. \quad (1)$$

[7] Although it is possible that a long-lived hedger will want to adjust his futures position after one period has passed, it is straightforward to show that an optimal multiperiod hedge holds the same futures position after each period. Thus, a long-lived hedger simply rolls over the entire position to the next period.

This initial position is non-negative, $x(S, D) \geq 0$, and is thus a long position. Symmetrically, for a hedger with endowment +E, the optimal initial futures position is $-x(S, D)$, a short position of identical magnitude. Therefore, all H hedgers take an initial futures position with magnitude $x(S, D)$. Of these hedgers, qH will be long-lived and roll over their position next period using spread futures, while $(1 - q)H$ will be short-lived and simply close their position next period.

An informed trader with conditional mean $+\theta$ optimally takes futures position $y(S)$ satisfying

$$\max_{y} \quad (\theta - S)y - \Gamma k\sigma^2 y^2/2 \tag{2}$$

if $S \leq \theta$. If the bid-ask spread is so large that $S > \theta$, then informed traders do not participate in the market, $y(S) = 0$. Similarly, an informed trader with conditional mean $-\theta$ optimally takes futures position $-y(S)$. Thus, $y(S)$ is the magnitude of the futures position for all I informed traders.

Alternatively, suppose that spread futures contracts are not available for trade. The same calendar spread trade can be made directly by trading in the underlying futures contracts, albeit with a different frictional cost, a bid-ask spread of S. The trades of both hedgers and informed can be inferred by setting D = S in the previous analysis. All H hedgers take an initial futures position with magnitude $x(S, S)$. All hedgers will close this position next period, but qH of the hedgers will also open the same size position in the subsequent delivery futures contract. As before, all I informed traders take an initial position of magnitude $y(S)$.

2. Competitive bid-ask spreads

This section considers the case of the bid-ask spread(s) for futures contracts being set competitively. Thus, the bid-ask spread is set so that market-makers break even in expectation. In the absence of spread futures, the breakeven condition in the primary futures market is

$$0 = H(1 + q)S \cdot x(S, S) + I(S - \theta) \cdot y(S). \tag{3}$$

Here, H hedgers generate trade volume $H(1 + q) \cdot x(S, S)$; market-makers receive the bid-ask spread S per contract traded. I informed traders generate volume $I \cdot y(S)$; market-makers receive the bid-ask spread S but expect to lose θ per contract traded due to information asymmetry. In the absence of spread futures, denote the competitive bid-ask spread in the primary futures market, satisfying (3), by S_{NSF}.

In the presence of spread futures, the breakeven conditions in the primary and spread futures markets are

$$\begin{aligned} 0 &= HS \cdot x(S, D) + I(S - \theta) \cdot y(S), \\ 0 &= HqD \cdot x(S, D). \end{aligned} \tag{4}$$

Here, H hedgers generate trade volume $H \cdot x(S, D)$ in the primary futures market, and $qH \cdot x(S, D)$ in the spread futures market; market-makers receive bid-ask spread S per primary futures contract traded and D per spread futures contract traded. I informed traders generate volume $I \cdot y(S)$ in the primary futures market; market-makers receive the bid-ask spread S but expect to lose θ per primary futures contract traded due to information asymmetry. In the presence of spread futures, denote the competitive bid-ask spreads in the primary and spread futures markets, satisfying (4), by S_{SF} and D_{SF}, respectively.

In order to eliminate the possibility that the adverse selection problem is so severe that it shuts down the futures market, it is assumed that $\theta \leq \Gamma\sigma^2 E$. By implying the existence of a (primary market) bid-ask spread large enough to eliminate informed trade, yet not eliminate all hedging activity[8], this guarantees the existence of bid-ask spreads sustaining keeping the markets open. This keeps with the focus of the paper, examining the introduction of calendar spread futures to an already existing futures market.

When the spread futures market is available, as long as it offers a lower transaction cost than direct trading, $D < S$, hedgers desiring a calendar spread will utilize spread futures rather than constructing the spread through simultaneous trading in the

[8] Specifically, any spread S satisfying $\theta \leq S < \Gamma\sigma^2 E$ suffices.

new and old futures contracts. Thus hedgers rolling over their old positions migrate to the spread futures market while hedgers initiating new positions as well as informed traders remain in the primary futures market. Furthermore, hedgers assessing their initial futures position recognize that their cost of rolling over their position later (if it becomes necessary) has dropped and therefore increase the magnitude of their initial futures position.

Since the traders migrating to the spread futures market are exclusively hedgers, the bid-ask spread in that market falls. In the model, since adverse selection generates the only trading friction, the bid-ask spread falls all the way to zero. Informed traders remain in the primary market; hedgers rolling over positions no longer use the primary market, while the usage from hedgers taking initial positions increases. Therefore, the hedging demand in the primary market may potentially either decline or rise, as may the equilibrium level of the competitive bid-ask spread there. The total hedging volume, encompassing trading in both the primary and spread futures markets, increases. Since each initial hedging position is expected to generate $(1 + q)$ round trips of trading, whether spread futures are available or not, and the number of potential hedgers is exogenously fixed at H, the total hedging volume $H(1 + q)x$ is effectively captured by x, the initial hedging volume per potential hedger. This intuition is formalized in Proposition 1. (The proofs of all Propositions are contained in the Appendix.)

Proposition 1. *The competitive bid-ask spread of the primary futures market may be increased or decreased by the introduction of calendar spread futures. The bid-ask spread is increased,* $S_{SF} > S_{NSF}$*, exactly when* $\theta > \Gamma\sigma^2 E\cdot[1 + \sqrt{(kH/I)}]\cdot(1 + q)/(2 + q)$. *The competitive bid-ask spread of the calendar spread futures is* $D_{SF} = 0$. *Total hedging volume is higher in the presence of spread futures, as* $x(S_{SF}, D_{SF}) \geq x(S_{NSF}, S_{NSF})$. *Hedgers are made better off by the introduction of calendar spread futures.*

In order to implement the new competitive bid-ask spreads with the introduction of calendar spread futures, it may be necessary to reduce the minimum tick size. In particular, if the minimum tick size in the primary market was set near the competitive bid-ask spread without spread futures, then the minimum tick size in the spread futures will need to be set below that of the primary market. Thus, it is natural to expect that the spread futures will be introduced with the reduced tick feature.

3. Monopolistic bid-ask spreads

The section considers the case of the bid-ask spread(s) for futures contracts being set monopolistically. The exchange is able to exercise monopoly power in setting the levels of the bid-ask spread(s). Furthermore, the bid-ask spreads are set to maximize the aggregate profit of the market-makers on the exchange (who may, for example, be the owners of the exchange). In the case without spread futures, the optimization problem is

$$\max_{S} \quad H(1 + q)S \cdot x(S, S) + I(S - \theta) \cdot y(S). \tag{5}$$

In the case with spread futures, the optimization problem is

$$\max_{D \leq S} \quad H(S + qD) \cdot x(S, D) + I(S - \theta) \cdot y(S). \tag{6}$$

The $D \leq S$ constraint is needed because if spread futures are not cheaper to trade, then traders can implement calendar spreads by trading directly in the primary market.

When a spread futures contract with lower transaction cost is available, as in the competitive case, traders sort themselves by market. Hedgers rolling over old positions use the spread futures market, while hedgers initiating new positions as well as the informed traders use the primary futures market. Again, the lower expected cost of rolling over positions later entices hedgers to take larger initial futures positions.

The presence of spread futures allows market-makers to price discriminate between hedgers and informed traders. Hedgers incur a cost S in taking their initial futures position and an additional cost $D < S$ if they roll the position over. Thus, the average trading cost for hedgers (including trading in both futures markets) is less than S, while the average trading cost for informed traders is S since they cannot effectively utilize spread futures. By price discriminating, market-makers can make trading more attractive for the desired hedgers and less attractive for the undesired informed traders.

Price discrimination will be most effective by emphasizing a large difference in average trading cost between hedgers and informed, increasing S and decreasing D. Optimal price discrimination sends D, the bid-ask spread in the calendar spread futures, all the way to its lower limit of zero, and increases S, the bid-ask spread in the primary market, above its level in the absence of spread futures. Price discrimination allows an overall lower expected transaction cost for hedgers, so the magnitude of hedgers' initial positions as well as the total hedging volume increase with the introduction of spread futures. This is formalized in Proposition 2. (An asterisk is used to denote monopolistic bid-ask spreads.)

Proposition 2. *The monopolistic bid-ask spread of the primary futures market is increased by the introduction of calendar spread futures,* $S_{SF}^* \geq S_{NSF}^*$. *The monopolistic bid-ask spread of the calendar spread futures is* $D_{SF}^* = 0$. *Total hedging volume is higher in the presence of spread futures, as* $x(S_{SF}^*, D_{SF}^*) \geq x(S_{NSF}^*, S_{SNF}^*)$. *Hedgers are made better off by the introduction of calendar spread futures.*

The monopolistic bid-ask spread in the primary futures market is determined by the tension between two opposing forces, the simultaneous desires to choose the bid-ask spread to maximize revenue from hedgers, and to minimize informed trade and its associated adverse selection problem. This tension exists whether or not calendar spread futures are present. The key variable is the average trading cost for hedgers; for example, this then determines the total hedging volume across the futures markets as well as the revenue market-makers receive from hedgers. Without spread futures, the average trading cost for hedgers simply equals the bid-ask spread in the primary futures market; with spread futures, it also includes the lower bid-ask spread of trading spread futures.

To maximize revenue from hedgers, the average trading cost for hedgers is set proportional to the hedger endowment size E (which essentially acts as the intercept of a linear demand curve in the presence of a monopolist), whether or not spread futures trade. To minimize adverse selection, the bid-ask spread in the primary market is set equal to θ, the informational advantage of informed traders. Without spread futures, this translates to an average trading cost for hedgers of θ. With spread futures, since hedgers do part of their trading in the lower-cost spread futures, this translates to an average trading cost for hedgers of less than θ. When E is large relative to θ, so the bid-ask spread is greater than the informational advantage of traders, and the adverse selection problem is eliminated, then the average trading cost of hedgers is identical with and without spread futures. Total hedging volume is then identical with or without spread futures. When E is smaller relative to θ, the average trading cost is set at a weighted average of the "maximize revenue" and "minimize adverse selection" levels. Therefore, the average trading cost for hedgers is lower with spread futures, leading to higher total hedging volume with spread futures.

Similar to the competitive bid-ask spread case, even if the minimum tick size was near the optimal bid-ask spread before the introduction of spread futures, in order to implement the optimal bid-ask spreads with calendar spread futures, it will be necessary for the spread futures to offer a reduced tick, relative to the primary futures market. Furthermore, since the optimal bid-ask spread in the primary market increases, only the spread futures, and not the primary market, need reduce its tick size.

Because the monopolist has an additional parameter to optimize over, the aggregate market-maker profit with spread futures, π_{SF}*, is always at least as large as the profit without spread futures, π_{NSF}*. The gain of the market-makers, $\Delta\pi^* = \pi_{SF}^* - \pi_{NSF}^*$ naturally depends upon the underlying parameters. The comparative statics with respect to the various parameters are given in Proposition 3.

Proposition 3. *Under monopolistic bid-ask spreads, market-makers gain by the introduction of calendar spread futures. The magnitude of the aggregate market-maker gain* $\Delta\pi^* = \pi_{SF}^* - \pi_{NSF}^*$ *is increasing in the number of hedgers* H, *the number of informed traders* I, *and the mean of information quality* θ. *The gain is decreasing in the size of hedging needs* E, *variance of informed quality* k*, trader risk aversion* Γ, *and risky asset volatility* σ.

From the market-makers' viewpoint, the introduction of spread futures trading allows price discrimination against the informed traders in terms of the bid-ask spread those traders face. Thus, the market-makers' gain depends upon the magnitude of the adverse selection problem, generated by the informed traders, that they face. This is increasing in the number of informed traders I, and the expected informational advantage θ of such a trader, and is decreasing in the noisiness of the information as measured by k. Similarly, since the magnitude of informed trade depends inversely upon the risk aversion Γ of informed traders and the underlying asset volatility σ, the market-makers' gain is decreasing in both of these parameters. The primary source of market-maker profit is the bid-ask spread received from trading against hedgers; thus the aggregate market-maker gain is increasing in the number of hedgers H.

Comparative statics with respect to E, the endowment size per hedger, is more complicated. As previously described, when E is large relative to θ, the average trading cost for hedgers with and without calendar spread futures is identical; the aggregate market-makers gain from the introduction of spread futures is zero. When E is smaller relative to θ, the average trading cost for hedgers is lower with spread futures, leading to a more significant difference in aggregate market-maker profits for the cases with and without spread futures. Thus, as E becomes smaller (or θ becomes larger), the aggregate gain of the market-makers from the introduction of spread futures becomes larger.

Of all these parameters, perhaps the two most likely in practice to determine whether a particular futures contract offers a relatively large magnitude gain for market-makers, and therefore an attractive opportunity for an exchange to innovate, are large values for H or I, corresponding to a large number of hedgers or informed traders, respectively. Note also that a large value of H generally translates to a large volume of hedging-based trading, while a large value of I generally translates to a large volume of information-based trading. Therefore, recognizing that designing and implementing a new security is a potentially expensive process, the introduction of a spread futures contract is most likely to make economic sense when the primary futures market already exhibits high volume. This is consistent with the observation that the spread futures introduced by the a/c/e to date are based on some of the most popular (as measured by volume) futures contracts traded on the Chicago Board of Trade.

4. Conclusion

Since a calendar spread futures contract is a derivative upon already available derivative securities, it does not offer hedgers the ability to hedge any risks that they cannot hedge already. Furthermore, it does not make it any easier to hedge those risks: spread futures should be no easier to trade than the futures underlying them. However, the introduction of spread futures allows partial separation between hedgers and informed traders. This allows competitive market-makers to charge a lower bid-ask spread to hedgers utilizing spread futures, effectively lowering the overall transaction cost for hedgers. Depending upon the sensitivity of hedging demand to the transaction cost, the competitive bid-ask spread in the primary futures market may either increase or decrease. Overall, transaction costs are more favorable for hedgers, leading to increased hedging and an increase in hedger welfare. Necessary to the implementation of this separation of traders is the smaller price increment, or reduced tick, allowed in the spread futures market: even if traders were able to negotiate both legs of the spread simultaneously by trading primary futures in the pit, the cost of so doing (as measured by the bid-ask spread) may be higher than trading the spread futures.

Similarly, when an exchange has the ability and desire to exercise monopoly power in setting bid-ask spreads to benefit market-makers, the introduction of spread futures will lead to a lower transaction cost in trading calendar spreads, and partial trading separation between hedgers and informed traders. The optimal bid-ask spread in the primary futures market will increase, but the other results are qualitatively similar to the competitive case: overall transaction costs are lower for hedgers, the amount of hedging increases, and hedger welfare is improved.

The principal insight here is that the presence of calendar spread futures trading allows the costs imposed upon the market-maker to be more directly borne by the traders who actually generate the cost. The adverse selection cost generated by informed traders is therefore able to be at least partially controlled by the introduction of spread futures trading. As this cost is ultimately imposed upon hedgers (and, in the case of monopolistically set bid-ask spreads, upon market-makers), hedgers (and market-makers) are made better off by their introduction. Futures contracts which already exhibit a large level of volume, such as those based on US government debts, are more likely to be good candidates for the introduction of spread futures trading.

Appendix

Proof of Proposition 1.

Optimizing (1), the initial futures position of a hedger with endowment –E is

$$x(S, D) = E - (S + qD)/\Gamma\sigma^2(1 + q) \qquad \text{if } S + qD \le \Gamma\sigma^2 E(1 + q),$$
$$= 0 \qquad \text{otherwise.}$$

Similarly, the initial futures position of a hedger with endowment +E is –x(S, D), so all hedgers take an initial position of magnitude x(S, D). In the case of no available spread futures,

$$x(S, S) = E - S/\Gamma\sigma^2 \qquad \text{if } S \le \Gamma\sigma^2 E,$$
$$= 0 \qquad \text{otherwise.}$$

Optimizing (2), the magnitude of the initial futures position for an informed trader is

$$y(S) = (\theta - S)/\Gamma\sigma^2 k \qquad \text{if } S \le \theta,$$
$$= 0 \qquad \text{otherwise.}$$

In the case of a competitive futures exchange with no available spread futures, aggregate market-maker profit $H(1 + q)S\cdot x(S, S) + I(S - \theta)\cdot y(S)$ equals

$$= H(1 + q)S(E - S/\Gamma\sigma^2) - I(\theta - S)^2/\Gamma\sigma^2 k \qquad \text{if } S \le \theta,$$
$$= H(1 + q)S(E - S/\Gamma\sigma^2) \qquad \text{if } \theta \le S \le \Gamma\sigma^2 E,$$
$$= 0 \qquad \text{if } S \ge \Gamma\sigma^2 E.$$

The competitive bid-ask spread S_{NSF} is in the first region and satisfies

$$(1 + q)S\cdot(\Gamma\sigma^2 E - S) = (I/kH)\cdot(\theta - S)^2. \qquad \text{(A1)}$$

(Although bid-ask spreads above $\Gamma\sigma^2 E$ generate zero profits, they do so trivially, as they also generate no trade.)

In the case with spread futures, the breakeven condition for the spread futures market, $HqD\cdot x(S, D) = 0$ implies $D_{SF} = 0$. Market-maker profit in the primary futures market $HS\cdot x(S, 0) + I(S - \theta)\cdot y(S)$ therefore equals

$$= HS[E - S/\Gamma\sigma^2(1 + q)] - I(\theta - S)^2/\Gamma\sigma^2 k \qquad \text{if } S \le \theta,$$
$$= HS[E - S/\Gamma\sigma^2(1 + q)] \qquad \text{if } \theta \le S \le \Gamma\sigma^2 E(1 + q),$$
$$= 0 \qquad \text{if } S \ge \Gamma\sigma^2 E(1 + q).$$

The competitive bid-ask spread S_{SF} is in the first region and satisfies

$$S\cdot[\Gamma\sigma^2E - S/(1 + q)] = (I/kH)\cdot(\theta - S)^2. \quad \text{(A2)}$$

In comparing (A1) to (A2), they share the same quadratic right-hand side (cost function) and range $0 \leq S \leq \theta$. The quadratic left-hand side (revenue function) of (A2) is the left-hand side (revenue function) of (A1) shifted horizontally to the right. To see this, substitute $S' = (1 + q)S$ into (A1) and compare to (A2). The left-hand sides of (A1) and (A2) intersect at $S = (1 + q)\Gamma\sigma^2E/(2 + q)$. If the shared cost function lies below the intersection of these revenue functions, then the revenue/cost intersection of (A1) occurs to the left of the revenue/cost intersection of (A2), which occurs to the left of the revenue/revenue intersection; that is, $S_{NSF} \leq S_{SF} \leq (1 + q)\Gamma\sigma^2E/(2 + q)$. If the right-hand side cost function lies above the intersection of the revenue functions, then the revenue/cost intersection of (A1) occurs to the right of the revenue/cost intersection of (A2), which occurs to the right of the revenue/revenue insection; that is, $S_{NSF} \geq S_{SF} \geq (1 + q)\Gamma\sigma^2E/(2 + q)$. The relevant condition reduces to whether $(I/kH)\cdot[\theta - (1 + q)\Gamma\sigma^2E/(2 + q)]^2$ is lesser or greater than $[(1 + q)\Gamma\sigma^2E/(2 + q)]^2$, which reduces to whether θ is lesser or greater than $[1 + \sqrt{(kH/I)}]\cdot[(1 + q)/(2 + q)]\cdot\Gamma\sigma^2E$.

To compare the hedging volumes with and without spread futures, note that with spread futures, $x(S_{SF}, 0) = E - S_{SF}/\Gamma\sigma^2(1 + q)$, and that without spread futures, $x(S_{NSF}, S_{SNF}) = E - S_{NSF}/\Gamma\sigma^2$. The relevant comparison is therefore S_{SF} against $(1 + q)S_{NSF}$. Transform (A1) by substituting $S' = (1 + q)S_{NSF}$ to get

$$S'\cdot[\Gamma\sigma^2E – S'/(1 + q)] = (I/kH)\cdot[\theta - S'/(1 + q)]^2 \quad \text{(A3)}$$

over the range $0 \leq S' \leq (1 + q)\theta$.

In comparing (A2) and (A3), they share the same quadratic left-hand sides benefit functions. The right-hand side cost function of (A3) is the right-hand side of (A2) shifted horizontally to the right. The revenue/cost intersection of (A3) occurs to the right of the revenue/cost intersection of (A2), that is, $(1 + q)S_{NSF} \geq S_{SF}$, implying that $x(S_{NSF}, S_{SNF}) \leq x(S_{SF}, D_{SF})$.

To calculate the hedger welfare change from introducing spread futures, first substitute $S = S_{NSF}$, $D = S_{NSF}$, and $x = x(S_{NSF}, S_{NSF}) = E - S_{NSF}/\Gamma\sigma^2$ into (1) to find hedger welfare without spread futures,

$$[(1 + q)\Gamma\sigma^2E - (1 + q)S_{NSF}]^2/2(1 + q)\Gamma\sigma^2 - (1 + q)\Gamma\sigma^2E^2/2.$$

Similarly, substitute $S = S_{SF}$, $D = 0$, and $x = x(S_{SF}, 0) = E - S_{SF}/\Gamma\sigma^2(1 + q)$ into (1) to find hedger welfare with spread futures,

$$[(1 + q)\Gamma\sigma^2E - S_{SF}]^2/2(1 + q)\Gamma\sigma^2 - (1 + q)\Gamma\sigma^2E^2/2.$$

Since $(1 + q)\Gamma\sigma^2E \geq (1 + q)S_{NSF} \geq S_{SF}$, the welfare change from introducing spread futures is non-negative. ♦

Proof of Proposition 2.

The initial futures position of hedgers x(S, D) and informed traders y(S) are given in the proof of Proposition 1. For the case without spread futures, aggregate market-maker profit from (5), $H(1 + q)S\cdot x(S, S) + I(S - \theta)\cdot y(S)$, equals

$$\begin{array}{ll} H(1 + q)S(E - S/\Gamma\sigma^2) - I(\theta - S)^2/\Gamma\sigma^2k & \text{if } S \leq \theta, \\ H(1 + q)S(E - S/\Gamma\sigma^2) & \text{if } \theta \leq S \leq \Gamma\sigma^2E, \\ 0 & \text{if } S \geq \Gamma\sigma^2E. \end{array} \tag{A4}$$

The first derivative over the region with non-zero trade is

$$\begin{array}{ll} H(1 + q)(E - 2S/\Gamma\sigma^2) + 2I(\theta - S)/\Gamma\sigma^2k & \text{if } S < \theta, \\ H(1 + q)(E - 2S/\Gamma\sigma^2) & \text{if } \theta < S < \Gamma\sigma^2E. \end{array}$$

The first-order condition implies the optimal spread $S_{NSF}*$ equals

$$\begin{array}{ll} [(1 + q)e + 2J\theta]/[2(1 + q + J)] & \text{if } \theta \leq e \leq 2\theta, \\ e/2 & \text{if } 2\theta \leq e, \end{array} \tag{A5}$$

where for convenience we define $J = (I/kH)$ and $e = \Gamma\sigma^2E$. Note that $\theta \leq e$ is the assumption previously made to guarantee that markets do not shut down from severe adverse selection.

For the case with spread futures, aggregate market-maker profit from (6),
$H(S + qD)\cdot x(S, D) + I(S - \theta)\cdot y(S)$, equals

$$\begin{array}{ll} H(S + qD)[E - (S + qD)/\Gamma\sigma^2(1 + q)] - I(\theta - S)^2/\Gamma\sigma^2k & \text{if } S \leq \theta, \\ H(S + qD)[E - (S + qD)/\Gamma\sigma^2(1 + q)] & \text{if } \theta \leq S \leq \Gamma\sigma^2E(1 + q), \\ 0 & \text{if } S \geq \Gamma\sigma^2E(1 + q). \end{array}$$

To optimize over $0 \leq D \leq S$, first note that for any $D > 0$, the same revenue with lower adverse selection costs can be achieved by raising S and lowering D to keep (S + qD) constant. Therefore, the optimal $D_{SF}* = 0$. Substituting back, it remains to maximize the following over S,

$$\begin{array}{ll} HS[E - S/\Gamma\sigma^2(1 + q)] - I(\theta - S)^2/\Gamma\sigma^2k & \text{if } S \leq \theta, \\ HS[E - S/\Gamma\sigma^2(1 + q)] & \text{if } \theta \leq S \leq \Gamma\sigma^2E(1 + q), \\ 0 & \text{if } S \geq \Gamma\sigma^2E(1 + q). \end{array} \tag{A6}$$

The first derivative with respect to S over the region with non-zero trade is

$$\begin{array}{ll} H[E - 2S/\Gamma\sigma^2(1 + q)] + 2I(\theta - S)/\Gamma\sigma^2k & \text{if } S < \theta, \\ H[E - 2S/\Gamma\sigma^2(1 + q)] & \text{if } \theta < S < \Gamma\sigma^2E(1 + q). \end{array}$$

The first-order condition implies the optimal spread S_{SF}* equals

$$\begin{array}{lll} (e + 2J\theta)/[2(J + (1 + q)^{-1}] & \text{if } \theta \le e \le 2\theta/(1 + q), & \\ (1 + q)e/2 & \text{if } 2\theta/(1 + q) \le e, & \text{(A7)} \end{array}$$

using the J and e notation as before.

The comparison of S_{NSF}* to S_{SF}* breaks up into three regions: $\theta \le e \le 2\theta/(1 + q)$, $2\theta/(1 + q) \le e \le 2\theta$, $2\theta \le e$.

For the region $\theta \le e \le 2\theta/(1 + q)$,

$$\begin{aligned} &2(J + 1 + q)[J + (1 + q)^{-1}](S_{SF}^* - S_{NSF}^*) \\ &= (J + 1 + q)(e + 2J\theta) - [J + (1 + q)^{-1}][(1 + q)e + 2J\theta] \\ &= qe - qeJ + 2J\theta q(2 + q)/(1 + q) \\ &\ge qe - qeJ + qeJ(2 + q) \\ &= qe(1 + J + qJ) \ge 0. \end{aligned}$$

For the region $2\theta/(1 + q) \le e \le 2\theta$,

$$\begin{aligned} &2(J + 1 + q)(S_{SF}^* - S_{NSF}^*) \\ &= (1 + q + J)(1 + q)e - (1 + q)e - 2J\theta \\ &= (1 + q)e(q + J) - 2J\theta \\ &\ge 2\theta(q + J) - 2J\theta \\ &= 2\theta q \ge 0. \end{aligned}$$

For the region $2\theta \le e$,

$$2(S_{SF}^* - S_{NSF}^*) = (1 + q)e - e = qe \ge 0.$$

Thus, for all parameter values, $S_{SF}^* \ge S_{NSF}^*$.

To compare the hedging volumes with and without spread futures, note that with spread futures, $x(S_{SF}^*, 0) = E - S_{SF}^*/\Gamma\sigma^2(1 + q)$, and that without spread futures, $x(S_{NSF}^*, S_{SNF}^*) = E - S_{NSF}^*/\Gamma\sigma^2$, so S_{SF}* and $(1 + q)S_{NSF}$* are compared. The previous three regions are considered.

For the region $\theta \le e \le 2\theta/(1 + q)$,

$$\begin{aligned} &2(J + 1 + q)[J + (1 + q)^{-1}][(1 + q)S_{NSF}^* - S_{SF}^*] \\ &= [1 + (1 + q)J][(1 + q)e + 2J\theta] - (1 + q + J)(e + 2J\theta) \\ &= (2 + q)qJe - 2qJ\theta + qJ^2\theta \\ &\ge (2 + q)qJ\theta - 2qJ\theta + qJ^2\theta \\ &= qJ\theta(q + J) \ge 0. \end{aligned}$$

For the region $2\theta/(1 + q) \leq e \leq 2\theta$,

$$\begin{aligned}
&2(J + 1 + q)[(1 + q)S_{NSF}^* - S_{SF}^*] \\
&= (1 + q)[(1 + q)e + 2J\theta] - (1 + q)e(1 + q + J) \\
&= 2(1 + q)J\theta - (1 + q)eJ \\
&\geq (1 + q)Je - (1 + q)eJ = 0.
\end{aligned}$$

For the region $2\theta \leq e$,

$$2[(1 + q)S_{NSF}^* - S_{SF}^*] = (1 + q)e - (1 + q)e = 0.$$

For all parameter values, $(1 + q)S_{NSF}^* \geq S_{SF}^*$, implying $x(S_{NSF}^*, S_{SNF}^*) \leq x(S_{SF}^*, D_{SF}^*)$.

Calculating the change in hedger welfare from introducing spread futures is similar to the calculation in Proposition 1. Since $(1 + q)\Gamma\sigma^2 E \geq (1 + q)S_{NSF}^* \geq S_{SF}^*$, the welfare change from introducing spread futures is non-negative. ♦

Proof of Proposition 3.

The aggregate market-maker profit π_{NSF}^* in the case of the monopolistic exchange, without spread futures, can be found by substituting (A5) into (A4). Thus, π_{NSF}^* equals

$$\begin{array}{ll}
(H/\Gamma\sigma^2)(-J\theta^2 + [(1 + q)e + 2J\theta]^2/4(1 + q + J)) & \text{if } \theta \leq e \leq 2\theta, \\
(H/\Gamma\sigma^2)(1 + q)e^2/4 & \text{if } 2\theta \leq e.
\end{array}$$

The aggregate market-maker profit π_{SF}^* in the case of the monopolistic exchange, with spread futures, can be found by substituting (A7) into (A6). Thus, π_{SF}^* equals

$$\begin{array}{ll}
(H/\Gamma\sigma^2)(-J\theta^2 + (e + 2J\theta)^2/4[J + (1 + q)^{-1}] & \text{if } \theta \leq e \leq 2\theta/(1 + q), \\
(H/\Gamma\sigma^2)(1 + q)e^2/4 & \text{if } 2\theta/(1 + q) \leq e.
\end{array}$$

The difference $\Delta\pi^* = \pi_{SF}^* - \pi_{NSF}^*$ gives the increase in market-maker profit from introducing spread futures. Thus, $(\Gamma\sigma^2/H) \cdot \Delta\pi^*$ equals

$$\begin{array}{ll}
(e + 2J\theta)^2/4[J + (1 + q)^{-1}] - [(1 + q)e + 2J\theta]^2/4(1 + q + J) & \\
= qJ[4\theta e(1 - J) - (2 + q)e^2 + 4J\theta^2(2 + q)/(1 + q)]/4(1 + q + J)[J + (1 + q)^{-1}] & \\
& \text{if } \theta \leq e \leq 2\theta/(1 + q), \\
& \\
(1 + q)e^2/4 + J\theta^2 - [(1 + q)e + 2J\theta]^2/4(1 + q + J) & \\
= (1 + q)J(2\theta - e)^2/4(1 + q + J) & \text{if } 2\theta/(1 + q) \leq e \leq 2\theta, \\
& \\
0 & \text{if } 2\theta \leq e.
\end{array} \qquad \text{(A8)}$$

First examine the first region, $\theta \le e \le 2\theta/(1 + q)$, where

$$\Delta\pi^* = (qHJ/4\Gamma\sigma^2(1 + q + J)[J + (1 + q)^{-1}]) \cdot [4\theta e(1 - J) - (2 + q)e^2 + 4J\theta^2(2 + q)/(1 + q)]$$

Examine the signs of a series of partial derivatives. First with respect to $\Gamma\sigma^2$ (note that Γ and σ^2 only appear together).

$$\begin{aligned}
&\partial (\Delta\pi^*)/\partial (\Gamma\sigma^2) \\
&= (qHJ/4(\Gamma\sigma^2)^2(1 + q + J)[J + (1 + q)^{-1}]) \cdot [-(2 + q)e^2 - 4J\theta^2(2 + q)/(1 + q)] \le 0.
\end{aligned}$$

Therefore, $\Delta\pi^*$ is decreasing in Γ and in σ. Next with respect to E,

$$\begin{aligned}
&\partial (\Delta\pi^*)/\partial E \\
&= (qHJ/4(1 + q + J)[J + (1 + q)^{-1}]) \cdot [4\theta(1 - J) - (2 + q)2e] \\
&\le (qHJ/4(1 + q + J)[J + (1 + q)^{-1}]) \cdot [4e(1 - J) - (2 + q)2e] \\
&= (qHJ/4(1 + q + J)[J + (1 + q)^{-1}]) \cdot [-2e(q + 2J)] \le 0.
\end{aligned}$$

$\Delta\pi^*$ is decreasing in E. Next with respect to θ,

$$\begin{aligned}
&\partial (\Delta\pi^*)/\partial\theta \\
&= (qHJ/4\Gamma\sigma^2(1 + q + J)[J + (1 + q)^{-1}]) \cdot [4e(1 - J) + 8J\theta(2 + q)/(1 + q)] \\
&\ge (qHJ/4\Gamma\sigma^2(1 + q + J)[J + (1 + q)^{-1}]) \cdot [4e(1 - J) + 4eJ(2 + q)] \\
&= (qHJ/4\Gamma\sigma^2(1 + q + J)[J + (1 + q)^{-1}]) \cdot [4e(1 + J + qJ)] \ge 0.
\end{aligned}$$

$\Delta\pi^*$ is increasing in θ. Next with respect to J,

$$\begin{aligned}
\partial (\Delta\pi^*)/\partial J &= (qH/4\Gamma\sigma^2(1 + q + J)^2[J + (1 + q)^{-1}]^2) \cdot \\
&\quad ([(2 + q)(2\theta - e)^2 + 4\theta^2(2 + q)/(1 + q)^2 - 4\theta e/(1 + q)]J^2 \\
&\quad\quad + [8\theta^2(2 + q)/(1 + q) - 8\theta e]J + [4\theta e - (2 + q)e^2]) \\
&\ge (qH/4\Gamma\sigma^2(1 + q + J)^2[J + (1 + q)^{-1}]^2) \cdot \\
&\quad ([(2 + q)(2\theta - e)^2 + 2\theta e(2 + q)/(1 + q) - 4\theta e/(1 + q)]J^2 \\
&\quad\quad + [4\theta e(2 + q) - 8\theta e]J + [2(1 + q)e^2 - (2 + q)e^2]) \\
&= (qH/4\Gamma\sigma^2(1 + q + J)^2[J + (1 + q)^{-1}]^2) \cdot \\
&\quad ([(2 + q)(2\theta - e)^2 + 2\theta eq/(1 + q)]J^2 + 4\theta eqJ + qe^2) \ge 0.
\end{aligned}$$

Since $J = (I/kH)$, $\Delta\pi^*$ is increasing in I and decreasing in k. Next, write $\Delta\pi^*$ as

$$\Delta\pi^* = (qI/4k\Gamma\sigma^2(1 + q + J)[J + (1 + q)^{-1}]) \cdot [4\theta e(1 - J) - (2 + q)e^2 + 4J\theta^2(2 + q)/(1 + q)]$$

and take the partial derivative with respect to H,

$$\begin{aligned}
\partial (\Delta\pi^*)/\partial H &= (qI/4k\Gamma\sigma^2(1 + q + J)^2[J + (1 + q)^{-1}]^2) \cdot (\partial J/\partial H) \cdot \\
&\quad ([J^2 + (2 + 2q + q^2)J/(1 + q) + 1][4\theta^2(2 + q)/(1 + q) - 4\theta e] \\
&\quad - [4\theta e(1 - J) - (2 + q)e^2 + 4J\theta^2(2 + q)/(1 + q)][2J + (2 + 2q + q^2)/(1 + q)])
\end{aligned}$$

$= (qI^2/4kH\Gamma\sigma^2(1 + q + J)^2[J + (1 + q)^{-1}]^2) \cdot$
$([4\theta^2(2 + q)/(1 + q) - 4\theta e] J^2 + 2[4\theta e - (2 + q)e^2] J$
$+ [-(2 + q)(2 + 2q + q^2)e^2 + 4\theta(3 + 3q + q^2)e - 4\theta^2(2 + q)]\cdot(1 + q)^{-1})$.

The coefficients of J^2, J, and $(1 + q)^{-1}$ are all positive. To show this for J^2,

$$4\theta^2(2 + q)/(1 + q) - 4\theta e \geq 2\theta(2 + q)e - 4\theta e = 2\theta qe \geq 0.$$

To show this for J,

$$2[4\theta e - (2 + q)e^2] \geq 2[2(1 + q)e^2 - (2 + q)e^2] = 2qe^2 \geq 0.$$

To show this for $(1 + q)^{-1}$ across the region $\theta \leq e \leq 2\theta/(1 + q)$, note that this term is quadratic in e, with a negative coefficient on e^2. Therefore, the minimum value over all possible values of e occurs at an endpoint, either at $e = \theta$ or $e = 2\theta/(1 + q)$. At $e = \theta$, the term has value $(2 - q^2)q\theta^2 \geq 0$, and at $e = 2\theta/(1 + q)$, the term has value $4\theta^2 q/(1 + q)^2 \geq 0$. Therefore, the term is non-negative over the region, and $\Delta\pi^*$ is increasing in H.

Now examine the second region, $2\theta/(1 + q) \leq e \leq 2\theta$, where

$$\Delta\pi^* = (1 + q)HJ(2\theta - e)^2/4\Gamma\sigma^2(1 + q + J).$$

It follows that $\Delta\pi^*$ is decreasing in E, increasing in θ, increasing in J (and therefore increasing in I and decreasing in k), and increasing in q. Taking the partial derivative with respect to $\Gamma\sigma^2$,

$$\partial (\Delta\pi^*)/\partial (\Gamma\sigma^2)$$
$$= [(1 + q)HJ/4(\Gamma\sigma^2)^2(1 + q + J)] \cdot [-2e(2\theta - e) - (2\theta - e)^2] \leq 0.$$

Therefore, $\Delta\pi^*$ is decreasing in Γ and in σ. Taking the partial derivative with respect to H,

$$\partial (\Delta\pi^*)/\partial H$$
$$= [(1 + q)(I/k)(2\theta - e)^2/4\Gamma\sigma^2(1 + q + J)^2] \cdot [I/kH^2] \geq 0.$$

$\Delta\pi^*$ is increasing in H. ♦

References

Allen, F., and D. Gale, 1994, *Financial Innovation and Risk Sharing*, MIT Press, Cambridge, MA.

Back, K., 1993, "Asymmetric Information and Options," *Review of Financial Studies*, 6, 435-472

Bakshi, G., C. Cao, and Z. Chen, 2000, "Do Call Prices and the Underlying Stock Always Move in the Same Direction?," *Review of Financial Studies*, 13, 549-584.

Buraschi, A. and J. Jackwerth, 2001, "The Price of a Smile: Hedging and Spanning in Options Markets," *Review of Financial Studies*, 14, 495-527.

Conrad, J., 1989, "The Price Effect of Option Introduction," *Journal of Finance*, 44, 487-499.

Detemple, J., and P. Jorion, 1990, "Option Listing and Stock Returns: An Empirical Analysis," *Journal of Banking and Finance*, 14, 781-801.

Duffie, D., and R. Rahi, 1995, “Financial Market Innovation and Security Design: An Introduction” *Journal of Economic Theory*, 65, 1-42.

Gray, R. W., 1970, "The Prospects for Trading in Live Hog Futures," in H. H. Bakken (ed.), *Futures Trading in Livestock-Origins and Concepts*, Chicago Mercantile Exchange, Chicago.

Johnston, E., and J. McConnell, 1989, "Requiem for a Market: An Analysis of the Rise and Fall of a Financial Futures Contract," *Review of Financial Studies*, 2, 1-23.

Kyle, A. S., 1984, “Market Structure, Information, Futures Markets, and Price Formation,” in G. G. Storey, A. Schmitz, and A. H. Sarris (eds.), *International Agricultural Trade: Advanced Readings in Price Formation, Market Structure and Price Instability*, Westview Press, Boulder, CO.

Longstaff, F. A., 1995, "Option Pricing and the Martingale Restriction," *Review of Financial Studies*, 8, 1091-1124.

Mayhew, S., 2000, "The Impact of Derivatives on Cash Markets: What Have We Learned?," working paper, University of Georgia.

Sandor, R. L., 1973, "Innovation by an Exchange: A Case Study of the Development of the Plywood Futures Contract," *Journal of Law and Economics*, 16, 119-136.

Silber, W. L., 1981, "Innovation, Competition, and New Contract Design in Futures Markets," *Journal of Futures Markets*, 1, 123-155.

Subrahmanyam, A., 1991, "A Theory of Trading in Stock Index Futures," *Review of Financial Studies*, 4, 17-51.

Working, H., 1953, “Futures Trading and Hedging” *American Economic Review* 43, 314-343.

Table 1. Monthly volume of reduced tick spread futures, by contract. Reduced tick spread futures were introduced in January 2001 for 30-year Treasuries, 10-year Treasuries, 5-year Treasuries, and 10-year Agencies, and introduced in May 2002 for 10-year Swaps.

	US Treasury Bonds	Ten-year Treasuries	Five-year Treasuries	Ten-year Agencies	Ten-year Swaps
Jan-01	212	110	0	0	N/A
Feb-01	25,750	74,050	25,756	8,117	N/A
Mar-01	11,410	21,196	9,300	0	N/A
Apr-01	11,178	1,696	0	0	N/A
May-01	79,438	160,178	36,786	3,000	N/A
Jun-01	24,260	59,588	41,272	0	N/A
Jul-01	774	1,298	0	0	N/A
Aug-01	147,530	221,256	84,920	2,452	N/A
Sep-01	33,940	156,135	26,008	2,276	N/A
Oct-01	8,920	1,322	4,874	304	N/A
Nov-01	101,800	189,346	79,612	3,558	N/A
Dec-01	24,356	58,204	54,320	992	N/A
Jan-02	1,838	2,020	900	0	N/A
Feb-02	146,946	233,906	177,262	2,388	N/A
Mar-02	133,184	163,432	54,970	0	N/A
Apr-02	300	4,006	150	0	N/A
May-02	230,174	380,750	234,304	286	7,702
Jun-02	96,836	158,966	102,608	0	0
Jul-02	8,740	32,578	676	0	0
Aug-02	219,074	423,324	177,700	0	2,502
Sep-02	116,844	191,996	52,898	166	8,314
Oct-02	11,562	11,136	752	0	0
Nov-02	272,150	588,564	416,586	0	23,524
Dec-02	125,444	260,522	125,730	0	0
Jan-03	15,262	66,586	11,624	0	0
Feb-03	355,406	778,548	473,828	0	20,480